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FORMAL AND ONTOLOGICAL ROUNDABOUTS

1. Introduction

Nowak's intellectual creativity and ability to develop his ideas systematically are among the strongest of his talents, as evidenced by his renowned theory of idealization. One wishes him the same success in his new scientific enterprise, that of unitarian metaphysics. In this paper I shall only consider a few formal and ontological (that is rational or categorical) aspects of metaphysics (Hartmann 1935; 1952).

To date, my exposure to unitarian metaphysics has been rather limited. I may therefore have missed something essential to it. Further discussion will help clarify any missing ideas or misunderstandings.

The history of Western science (inclusive of ontology, as one of the sciences) shows that the passage from any local theoretical framework (i.e. from the theory of some kind of phenomenon – physical, chemical, economic, legal or whatever), or from general methodological theories (as for the idealizational understanding of science) to metaphysical (ontological) ones, is beset by difficulties. In fact, metaphysics magnifies any possible categorical weakness present in the source theories.

Insofar as unitarian metaphysics is (also) a generalization of the idealizational understanding of science, it prove that the latter suffers from a number of deficiencies. Unfortunately, however, unitarian metaphysics seems to have problems of its own.

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2. A Few Basic Data

Let me summarize the basic steps of unitarian metaphysics. Let **X** be an object, and **P** a set of attributes (properties); **P** is subsequently divided into two subsets, **S** and $\overline{\mathbf{S}}$. **S** is the space of attributes of **X**: each point of **S** represents an attribute possessed in some degree by **X**. $\overline{\mathbf{S}}$ is the set of attributes not possessed by **X**. Basically, $\overline{\mathbf{S}}$ is the complement of **S** with respect to **P**. $\overline{\mathbf{S}}$ may therefore be represented as **P** − **S**. Attributes pertaining to the set **S** are termed positive, and attributes pertaining to **P** − **S** are termed negative (cf. Nowak 1997). We shall shortly see that a serious problem lurks in this apparently unproblematic setting (see section 4 below).

Objects and attributes are not on a par. The latter, in fact, play a deeper and more substantial role than the former. Attributes are the basic building blocks of unitarian metaphysics, whereas objects are subsequent and derived from attributes. In short, objects are collections of attributes.

So far, the theory's general framework is classical enough. Its underlying formal model is basically set theoretic, with points understood as attributes. An object is a bundle (set) of attributes. Basic attributes ("points") are simples, by which is meant that they are not composed of other attributes. The only serious departure from other set-theoretic understandings of metaphysics is the further requisite that points (basic attributes) should vary in intensity. Different objects are therefore distinguished not only by different attributes but by the intensity (degree) of their attributes as well. Objects with the same attributes but different degrees of intensity are different objects.

The point is relevant because intensity is the crucial aspect of Nowak's understanding of modalities. Possible objects, in fact, are given by variations of intensity. More precisely, an object $\boldsymbol{X'}$ is (merely) possible relative to **X** if (1) both **X** and $\boldsymbol{X'}$ share the same set **P** of attributes, and (2) $\boldsymbol{X'}$ possesses at least one attribute to a different extent from **X**. Possible worlds are collections of objects grounded on the same set of attributes and distinguished by the latter's different intensities. A clear understanding of the idea of intensity is therefore mandatory for precise development of the modal part of the theory.

3. Intensities

Given Nowak's otherwise classical formal framework, I confess I am bewildered by the above idea of simples that can vary. I do not see how Nowak can harmonize this aspect of his theory with his formal framework. According to the latter, all the variability one may think of requires and depends on some structure, i.e., it requires and depends on some complex. I am not against the idea of thick atoms, which is the idea of basic elements ("simples") endowed with a rich structure. Recent formal developments, as in the case of synthetic differential geometry, provide substance for the claim. Among the many outcomes of synthetic differential geometry, it will be recalled that a new, original and highly interesting theory of the continuum derives therefrom. There is nevertheless a price to be paid (providing it is a price), namely the failure of the principle of the excluded third.[1]

A different and much easier way out is to consider intensities as atoms, and attributes as types (sets or classes). Thus, attributes become collections of intensities, and the difference between attributes and objects is that the latter are collections constituted of atoms, so that every atom belongs to a different class of intensities (each is the representative of its equivalence class). This picture is thoroughly set theoretic. On the other hand, Nowak's explicit claim that attributes are simples seems to rule out this interpretation. As a matter of fact, one discerns an underlying indecision between the two interpretations that I have put forward.

In the rest of the paper, I shall no longer consider the problem of the intensity of being. Henceforth, attributes will be interpreted as basic items, with no internal variation.

[1] Bell (1998) is a philosophically oriented elementary introduction to synthetic differential geometry. Schuster, Berger and Osswald (2001) explores new formal theories of the continuum. The thin-thick terminology is due to Albertazzi (2002). The ontological and formal problem of thick points and its link with process semantics are discussed by Poli (2002).

4. Negatives

Returning to the other main points of the theory, I am also perplexed by the concept of a "negative attribute." Providing that my understanding is correct, the concept of a "negative attribute" appears to be interpreted in two different ways. According to the first interpretation, a negative attribute is an attribute which an object does not possess. According to the second interpretation, a negative attribute is the complement of a positive attribute. The two interpretations are patently different. The former claims that an object lacks an attribute, the latter that an object possesses an attribute, which is the complement of another attribute. The understanding of "negative" seems to waver continuously between the two readings.

Further analysis of the second of the just mentioned interpretations might be helpful. In short, the "complement" interpretation claims that the difference between positive and negative attributes is the same as the difference between concave and convex, or between a temperature of say 15° above or below zero. Both cases (the so-called positive and the so-called negative) are perfectly determinate and are termed positive or negative only (well, mainly) by convention. The distinction between conventionally positive and conventionally negative attributes can be traced back at least to Kant (1763) and does not raise any major ontological problem. In fact, the above separation between a positive and a negative side of attributes is simply not well-grounded from an ontological viewpoint: in both cases one is dealing with one and the same attribute, seen from different viewpoints or considered in respect to a conventionally established point-zero. The conclusion is therefore that in such cases there is one and only one attribute (call it temperature or curvature or whatever).

Unitarian metaphysics does not admit more than one actual object with the same attributes. We are therefore forced to conclude that two objects composed of the same set of attributes can be distinguished only if the degree (intensity) of at least one attribute is different (thus distinguishing actual from possible objects) or if at least one attribute is positively possessed by one object and negatively possessed by the other.

From what we have just seen, it is clear that this interpretation of attributes views them as types (or classes) whose elements may be divided into two subclasses, termed positive and negative. I have already pointed out that unitarian metaphysics does not seem willing to accept the idea of attribute as class. We have nevertheless found different paths in that direction.

The troubles encountered by the interpretation of a negative attribute as complement of a positive one suggest reverting to the other interpretation, i.e. to the idea of negative attribute as lacking attribute. The idea of considering objects not only from the point of view of the attributes that they have, but also from the point of view of those they lack seems promising.

Let us explore the following hypothesis:

> **Thesis of Maximality**
> Objects are maximal entities. Given the set **P** of the world's attributes, any object is composed of all attributes, either in positive or in negative.

Let us consider the following trivial example. Let **P** be $\{a, b\}$. The world grounded on **P** may contain four non-equivalent objects: ab, $a\bar{b}$, $\bar{a}b$ and $\bar{a}\bar{b}$. ab is the top, what Nowak calls "transcendental object." It is positively composed of all the world's attributes. $\bar{a}\bar{b}$ is the bottom, the null object, the maximum reduct, the object negatively composed of all the world's attributes. $a\bar{b}$ and $\bar{a}b$ are, let's say, *normal* objects, that is, objects composed by both positive and negative attributes.

A few comments are in order. If the above interpretation is correct, negative attributes do not play any role whatever. Inclusion, as customary, is governed by positive attributes alone. The only exception is bottom, understood as an element of everything. It is well known that accepting the bottom simplifies calculation and helps generalization. If its metaphysical legitimacy is nevertheless questioned, other choices are available, as for mereology. Trivial though it may be, the example is valuable in that it provides needed evidence in favor of a conclusion that is not apparent in Nowak's writings: the formal structure underlying unitarian metaphysics is lattice theory.

5. Holes and Other Vagaries

From the above conclusion, it follows that the extensive talks about ontic holes (Paprzycka 2000) lose much of their interest, as it should be. In fact, there is nothing negative in **P** – **S**. It is a set like any other. Moreover, bundles do not have holes. The latter arise only if more complex structures are considered.

A little Aristotle may be of help here. The idea of ontic holes, or lacks, may require what Aristotle called "by nature." An object may only lack something that by nature it should possess and for some reason it actually does not have. Humans do not lack the attribute of being iron-made because being iron-made is not part of their nature. However difficult it may be to set out the concept of "by nature" precisely, it nevertheless provides a hint as to where the problem lies.

All the problems discussed thus far converge on the question as to which formal framework is best suited to ontological analyses.

6. Dynamics

The problem addressed by the previous section calls for a theory of both wholes and of levels of reality. The "by nature" requisite, in fact, requires a given structure (whole) at a certain level of complexity and energy. Aristotle discussed the "by nature" requisite in his theory of wholes. The questions concerning the theories of wholes and their parts and of the levels of reality are highly intricate and I cannot deal with them in any detail here (but see Poli 2001ab). I restrict myself to making the following point. A level of reality may comprise different kinds of dynamics. One may subsequently distinguish between (a) dynamics relative to the *unfolding* of reality, i.e. those relative to processes internal to a stratum of reality which lead to realization of its possibilities, and (b) dynamics relative to the *potentiation* of the level, i.e. those relative to processes among strata of reality by which a higher level emerges from a lower one. The terminology is very close to that adopted by Nowak himself. *Prima facie*, I do not regard this as a mere lexical coincidence. On the other hand, a few scattered notes by Nowak seem to imply that he has trouble with the theories of both wholes and of levels of reality and prefers to follow a different route. Not having enough information, I am forced to leave the situation unresolved.

7. Codimension

We saw earlier that elementary algebra provides the basic formal structure of unitarian metaphysics. For my part, I simply do not believe that lattices are basically everything that metaphysics has in store.

As soon as the general framework is made more flexible, new possibilities emerge. Let us seriously maintain that **P** is a *space* of attributes. The latter may therefore be considered the dimensions of **P**. In every space – let us say of dimension n – sub-spaces of smaller dimension are discernible. For instance, a three-dimensional space comprises not only three-dimensional objects but also two-dimensional objects (planes), one-dimensional objects (lines), and zero-dimensional objects (points). This much is obvious. Let us now relativize to a base the concept of dimension. One may therefore say that a plane is a 2 dimensional base in a 3 (or more) dimensional space. We define as the codimension of a base the difference between the dimension of the environment space and the dimension of the base. In Nowak's terms, positive attributes form the base's (the object's) dimension and negative ones form its codimension. If the base consists of a plane in a three-dimensional environment space, its codimension is 3 – 2 = 1. If instead the base is a line in a three-dimensional environment space, its codimension is 3 – 1 = 2.

The important point is that there are situations in which the co-dimension of a base yields fundamental information, more important than the information provided by its dimensions. The following elementary example may be helpful.

Consider a geographical map (i.e. a two-dimensional model) and assume that we wish to cross a boundary on it. In geographical maps, boundaries are drawn as lines – that is, they are one-dimensional objects. In this case, the codimension of the event "crossing a boundary" is given by 2 – 1 = 1. The same situation can be modeled in more concrete terms. Let the environment space now be a three-dimensional physical region so that crossing a boundary is like passing through a door – that is, a two-dimensional surface. In this case too – and this is the interesting point – the codimension of the phenomenon "crossing a boundary" is the same, because we have 3 – 2 = 1. If we then add a temporal dimension, the environmental space becomes four-dimensional, while the boundary becomes three-dimensional (the two dimensions of the door plus time). And once again the crossing is one-dimensional: 4 – 3 = 1. This radical and rather crude example (from which many technical details have been omitted, for example exact specification of what a base is) shows that what characterizes a considered phenomenon is not the dimension of the environment space, nor that of the relevant sub-space, but the difference between their dimensions. Given that what remains constant in the multiplicity of constructable models is the codimension of the phenomenon, we may ignore the specificity of models and concentrate on the codimension of the phenomena under scrutiny.

From an ontological point of view, this finding strikes me as of considerable importance. It tells us that the phenomena being modeled display symmetries (or, if one prefers, invariants) which are independent from many positive details of its models. Negative information may be deeper than any positive information (but note how seriously misleading the terms 'positive' and 'negative' are!). At bottom, the above example tells us that objects have an intrinsic hardness and are therefore not the fruit of our projections.

8. Adjoints

"Reductions" and "transcendentalizations" are straightforward procedures: minus one or plus one attribute, and iterations thereof. Apart from terminological infelicities, the idea behind them is clear enough. I think it is a basically correct idea. Unfortunately, Nowak's reliance on elementary algebra has the effect of trivializing the idea. To make its yield apparent we are compelled to adopt a richer environment.

The mathematical theory of categories may provide a way out. Let me consider only the simplest example, namely the connection between the category of topological spaces (**T**) and the category of sets (**S**). These two categories may be linked by a functor, going from spaces to sets, **F**: **T** → **S**. The functor **F** is called a *forgetful* functor, because it “forgets” structure. Given any topological space $\mathbf{T}^*$, the forgetful functor applied to $\mathbf{T}^*$ gives its underlying set, that is, the collection of its points deprived of their opens. Rewriting what I have just said in Nowak’s own terminology, any application of a forgetful functor provides the reduct of a structured collection of items. The question now becomes: what about transcendentalization? From a mathematical viewpoints, Nowak’s transcendentalizations are cases of *lifting* functors, i.e. of functors adding structure. It is easy to realize that the problem now becomes: which structures should be added?

The great advantage of category theory is that it studies *connections between functors*. To cut a long story short, forgetting and lifting functors are connected by special relations (called ‘adjoints’). More precisely, as far as our example is concerned, the forgetful functor between **T** and **S** has both a left and a right adjoint: the former assigns to each set its discrete space, the latter its co-discrete space. In other words, the former assigns to a set **S** its most disconnected space, the latter its most connected space. This proves that there may be different lifts (transcendentalizations, in Nowak’s terms) from a set to the spaces based on it.

Adjunctions are everywhere in mathematics. To provide a couple of logically oriented examples, quantifiers and modalities can both be seen as cases of adjunction.

The situation may sometimes be iterated, adding more and more structure to initial items or forgetting more and more structure from complex items. Further details require rather heavy technicalities, because compatibility conditions (called coherence conditions) among the various operations should be properly specified. Technicalities aside, what really matters is that category theory provides the formal environment for studying the relationships between reductions and transcendentalizations.

Contemporary mathematics provides the tools for exploring the characteristics of the far from trivial operations of passing from simple to complex structures, and vice versa. Unitarian metaphysics (and, for that matter, the methodology of idealization as well) is still mainly based on trivial mathematics. The problem is not that elementary algebra is wrong. Obviously, it is not. The point is that elementary algebra is too poor and too rigid a tool for ontological analysis. In fact, I see no reason for holding that the basic structures of reality rely on or instantiate the simplest aspects of elementary algebra. Full-fledged ontology requires more powerful and more flexible conceptual and formal environments.

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