XI (APPENDIX)
PROCESS SEMANTICS

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Abstract

The adverbial version of process semantics advocated by Rescher is discussed. The limitations of the logical approach requires a reconceptualization of process semantics. The new interpretations are tested against the problems of (1) dynamic families, (2) dynamic identities, (3) ontology of processes and their mathematical models.

1. Introduction

The various chapters of *Process Metaphysics* present and carefully discuss a number of basic ontological categories. The Appendix to the volume, "Process semantics", is intended to provide a formal development of some of the categories previously discussed. Strangely enough, the topics addressed in the Appendix are substantially different from those discussed in the previous chapters.

The focal problem of a process semantics, according to Rescher, is that of non existent entities. In his words: "How can there possibly be a namable, identifiable, discussible individual – such as the winged horse Pegasus – that does not actually exist?"[1]

Besides failing to explain why non-existents should be the main problem for the development of an acceptable process semantics, the solution proposed by Rescher does not solve the problem at hand.

Rescher's first step is the idea that, logically speaking, talking about something is to claim its reality. As a matter of fact, mainstream logical

orthodoxy interprets the structure $F(a)$ – meaning: the item a has the property F – as $\exists x(x = a \wedge F(x))$ – that is: there is an entity x in the universe of discourse identical with a and x has the property F. The well-known answers advanced by Russell and Quine in the first half of the past century are then considered. Surprisingly, the more recent way out offered by free logics – what can nowadays be called the *standard solution* to the above problem – is not even mentioned. In a sense, Rescher's silence is justifiable, because a philosophically acceptable solution of the problems of reference requires an answer deeper than recourse to partial functions, the solution offered by free logics.

Many different kinds of non-existent entities, in fact, can be objects of our referential acts. Synoptically:

- Past and future entities obviously do not exist. At most they existed and will possibly exist.
- Mental (psychological) and social entities do not exist in the same way as material entities are said to exist.
- Ideal entities do not exist, because they are not temporally bounded like truly 'real' entities are.
- A much trickier case concerns nominalized predicates. To what do they refer? Let us consider this last case in some detail.

2. Nominalizations

Classically speaking, the subject of a given proposition denotes an object, i.e. something that is independently saturated, ontologically self-sufficient and complete. The predicate is instead something that is intrinsically unsaturated and which requires the former to acquire completeness. Predicates correspond to concepts. By means of nominalization we obtain a situation of the type $P(F)$, where the original predicate appears in the guise of a noun in the subjective position. In this case, F is no longer unsaturated as it was in $F(a)$, but it has the same subjective characteristics as a, except that corresponding denotatively to a is an entity in the universe of discourse. May we therefore say that also corresponding to the predicate F is an object of the same type as the one that corresponds to a? Obviously not. Frege says that corresponding to a nominalized predicate is a *conceptual correlate* which has individual value and is therefore saturated. Between the predicate and the nominalization of the predicate, i.e. between 'F' and 'the concept F' to use Frege's expression, there is a

relationship of *representation*. That is to say, in this situation 'the concept *F*' becomes the *individual* representative of '*F*', where the latter stands as the argument and the former as the value of the representation function.

Objects denoted by nominalized predicates are *intensional entities*, or in other words, properties and relations which have their own abstract form of individuality. We thus find ourselves in a situation where there are objects and conceptual correlates endowed with their own specific individuality, as opposed to a lack of individuality by concepts. A concept as such, in that it lacks individual characteristics, cannot be part of any universe of discourse. With the nominalization of *F* in the sense of 'the concept *F*' one obtains an individual term in the theory of concepts which denotes not concepts but special objects.[2]

From a formal point of view, the theory of nominalized predicates can be developed in different ways. Some reasonable options are the following:

(Abelard =)	$[\forall F][\neg\exists x](\mathrm{F} = x)$
(Abelard ≡)	$[\forall F][\neg\exists x](\mathrm{F} \equiv x)$
(Plato =)	$[\forall F][\exists x](\mathrm{F} = x)$
(Plato ≡)	$[\forall F][\exists x](\mathrm{F} \equiv x)$

where "≡" is used as the sign for indiscernibility, *i.e.*, $a \equiv b =_{\mathrm{df}} \forall F[F(a) \leftrightarrow F(b)]$.

The two Abelard options claim that nominalized predicates are singular terms that fail to refer (to any *thing*). The two Plato options claim that nominalized predicates refer to entities of the universe of discourse. Cocchiarella (1986) proved that the connections between the various theses above are as follows:

- (Abelard ≡) implies (Abelard =), that is to say: *indiscernibility implies identity*;
- (Plato =) implies (Plato ≡), that is to say: *identity implies indiscernibility*.

It is worth noting that Russell's antinomy requires (Plato =). Applied to $\exists \mathrm{F}\forall x(\mathrm{F}(x) \leftrightarrow \exists \mathrm{G}[x = \mathrm{G} \wedge \neg \mathrm{G}(x)]$, Russell's argument shows that the assumed concept is a non-thing, i.e., $\forall x(\mathrm{F}(x) \leftrightarrow \exists \mathrm{G}[x = \mathrm{G} \wedge \neg \mathrm{G}(x)]) \rightarrow \neg\exists x(\mathrm{F} = x)$.[3] The phenomenon of nominalization forces us to recognize

how sophisticated must be the framework for developing a formally acceptable theory of reference.

3. Adjectives and adverbs

From an ontological viewpoint, all the above problems have been mainly discussed by relying on the underlining category of 'object'. Process metaphysicians claim that at least some of the many problems arising within classically-oriented theories derive from their object-orientation, and that a process-orientation would offer better and easier solutions.

In order to test this alternative viewpoint, Rescher develops Quine's adjectival theory of predication. According to Quine, the problem of nonexistents might be solved by substituting named individuals with suitable adjectives. In this way, singular terms are transformed into predicative expressions. This solution has the advantage of remaining within the boundaries of first-order logic, whereas the above-presented discussion of nominalization requires a higher-order framework. On the other hand, it presents its own problems. In fact the "elimination of substantives in favor of adjectives does not … involve the actual elimination of objects to which such ex hypothesis noninstantiated predicates are (nominally) attributed".[4] Therefore, Rescher proposes to take a further step by "trading *adjectives* for *adverbs*".

The move advocated by Quine requires the translation from "Pegasus" to "pegasizer". Rescher's move requires the further transformation to "pegasizes". The subsequent question is unavoidable: What (if anything) pegasizes? According to Rescher, the required "it" ranges on spatiotemporal locations.

Linguistically, the objectual expression "Pegasus" has been replaced by the processual expression "pegasizes". Categorically, the individual substance as transcendent bearer of its determination has been substituted by a location. What pegasizes is a spatiotemporal location.

This solution of a clearly Brentanian flavour[5] runs nevertheless into serious difficulties. Let us consider two of them.

Firstly, if what "*x*-es" occupies a spatiotemporal volume, what about the points of the volume? Shall we accept that each of its points pegasizes? If we detatch one point, does it continue pegasizing? And what about the remainder? No obvious solution seems to emerge.

Secondly. Suppose that no spatiotemporal volume of our model pegasizes. This is as it should be: as a matter of fact, in our bold reality

nothing pegasizes. But then, how can we talk about (the idea of) pegasizing? It seems that we have gone back to the very same problem encountered by the mainstream logical framework from which we started.

The problem of non-existents may therefore not be the best entry point for consideration of the pros and cons of a process-oriented semantics.

Moreover, and to my great regret, I fear that the twentieth-century forms of logic may not be the most suitable environments for developing a proper process semantics. Something more radical is needed than switching from substantives to adverbs.

Let us start anew, reconsidering from the very beginning the problem of process semantics from scratch.

4. Types of process semantics

To my understanding, "process semantics" may be understood in at least two different ways. On one view, process semantics is a *kinematics*, the study of *transitions* from state A to state B (also in the plural: from states As to states Bs). The past fifteen years have seen enormous interest in this kinematic understanding of process semantics, mainly driven by the needs of computer science and formal linguistics, and a number of different formal frameworks have been proposed for it.[6]

For the philosophically minded reader, the less demanding formalism is possibly the one based on a generalization of possible world semantics. Basically: the usual modal interpretation of possible world semantics adopts frames with *one* relation of accessibility between worlds; the temporal interpretation based on Prior's early insights requires frames with *two* relations of accessibility; the processual or dynamic interpretation requires frames with *n* relations of accessibility. The latter versions of possible world semantics are also known as poly-modal Kripke models or labeled transition systems. Seen thus, process semantics are a generalization of the logical perspective linked to the names of Russell, Tarski, Carnap, Hintikka and Kripke.

Technically speaking, the novelties introduced by the kinematic rendering of formal logic are wide and deep. In fact, the dynamicization of classical (and intuitionistic) logics poses new and interesting problems. On the other hand, it is interesting to note that the *language* of transition logics is basically the language of the 150-year-old relation algebras.[7] Many results are new, the concrete systems under analysis and the reasons for

studying them are new as well, but the general framework has been known for ages.

From the point of view of process metaphysics, the above 'mild' interpretation of process semantics only scratches the surface. In fact, none of the real problems lying behind properly understood process semantics is considered. The problems faced by a thoroughly *processual* interpretation of process semantics are much more profound than those addressed by the state-transition interpretation so far considered. In one sentence, the basic difference between the two interpretations is more ontological than formal. Figure 1 provides outline of their main difference.

Figure 1

State-transition interpretation Processual interpretation

According to the state-transition interpretation, the basic ontological items are states (represented by the two solid black spots on the left). Processes connect states; they mark the passage from state to state (dotted line). Thoroughly processual interpretations read the situation the other way round: processes are the basic ontological items (the thick black line on the right). Sometimes, they discern a pause in something that may appear to be static items (dotted circles).

From an ontological viewpoint, some sort of gestaltic switch takes place between the two interpretations: the one considers to be basic what the other considers to be derived, and vice versa.

The first interpretation is deeply rooted in the history of Western philosophy and science. Its basic conceptual apparatus is well developed and has been widely tested, and is now firmly ingrained in our common sense understanding of the world. The conceptual apparatus of the other interpretation is much less developed, for a number of apparently good reasons.

The general situation may be summed up in the following thesis: either the processual framework is a weird variant of the basically sound state-based ontology, or it is something like the dawn of a new vision. Those who feel uneasy with such a bold claim may perhaps gain reassurance from the history of Western science: Galilean physics in its early days was precisely something of the sort. The underlying idea is that a thoroughly processual framework may pave the way for a non-

exclusively physically oriented vision of science (and of philosophy, as one of the sciences, as well).

If this is even only partially true, the lack of a completely developed conceptual framework for dynamic ontology is understandable.

Be that as it may, three problems at least are apparent:

1. A proper conceptual development of both frameworks is required (the requirement is obviously much more demanding for the processual one, as said);
2. The development of adequate formal models for any of the two frameworks is required as well;
3. Systematic comparisons between the various models should be developed to bring out their hidden or unexpected features (it may even turn out that the differences between the two viewpoints is less striking from a formal point of view than from an ontological one).

In what follows, I shall focus on problems that any process semantics may sooner or later have to face.

5. Dynamic families

Most systems derive from the generalisation of study of a single type of movement: that of the planets, or, if one prefers, that of a material point. Mathematically speaking, the simplest model of a changing situation is that of an attractor defined as the minimum in a basin of attraction. The most obvious depiction of this model is that of a sloping hillside and a ball which rolls down it until it reaches the bottom. By obvious analogy, I shall call this model the ‘skier model’.

The skier model has been enormously helpful in modeling phenomena of the most disparate kinds. Moreover, it can be easily extended to more and more complex situations (i.e., by admitting a dynamics of the attractors too). Nonetheless, the skier model seems unable to account for many phenomena (cognitive ones among others).

Another type of model may be helpful in these more complex cases. This other model I shall call the ‘surfer model’.

In the surfer model, the dynamic item is ‘balanced’ on the wave and moves by following the wave. In this case the dynamic item does not converge toward the minimum of potential.

In the skier model, the dynamic item follows a family of possible trajectories determined by the slope. Due to the presence of a gradient, the item 'falls' towards the minimum. In the surfer model, there are in fact *two* dynamic items: the wave which 'carries' the surfer, and the surfer himself as he strives to keep his balance on the wave. Unlike the case of the skier, in that of the surfer there is no pre-existing slope characterized by a particular minimum of potential. The context is instead that of a succession (a flow) of waves of a certain intensity. This flow enables *another item* (the surfer) *to be transported* in a situation of *local equilibrium*.

The former case (skier) starts with a dynamics characterized by the presence of a minimum of potential. The latter (surfer) starts from two stratified dynamics (one dynamics 'carries' the other) characterized by a situation of local coupling.

In order to have a label, I shall refer to the surfer dynamics as a 2-dynamics (short for two-dimensional dynamics). It requires only a moment's thought to realize that many cognitive phenomena are characterized by a 2-dynamics.[8]

Of course, the hypothesis that there may be phenomena like 2-dynamics enables us to formulate further hypotheses immediately. One may enquire, for example, whether there are phenomena characterized by 3-dynamics and what they are. And what about *n*-dynamics (where n = 1,2,3,…)?

1-dynamics have proved successful in modeling the dynamics of material points. 2- and *n*-dynamics may prove to be necessary for modeling the dynamics of ontologically more complex items.

6. Dynamical identities

To render the discussion more explicit, we may consider an adequately complex centre of action like, for example, a psychic agent. The essential difference between this example and the cases considered so far is that a psychic agent is a centre of action which has a presentation of itself and of its environment. Note that it is not necessary to assume that these presentations are explicit or linguistic.

Given this situation, it is evident that various forms of identity, structured in several dimensions, are systematically active for every centre of action. We may distinguish these dimensions as, on the one hand, the relationship between actual and potential identity, and on the other, the relationship between internal and external identity.

Actual external identity is governed by the interactions of the centre of action with either the other centres of action in the environment or the environment itself. On the other hand, actual internal identity is the specific vision that the centre of action possesses of itself. There is no reason to suppose that these two forms of identity must coincide. Their differences, and the tensions that spring from them, plainly generate a specific psychic dynamics. Besides the presentation that the centre of action has of itself, and besides the presentations that other centres of action (if any) have of the former, a satisfactory depiction of the situation must also consider the presentations that any centre of action has of the other centres. It should also be noted that the various centres of action's presentations of the future modulate the coupling together of the actual identities and the dynamics generated by them.

Perhaps the crucial aspect here is the subtle dialectic between actual and potential indicated by the presence of these various identities.

To sum up, we can distinguish two families of identities, which I shall call families of 'direct identity' and of 'identity of presentation'. Direct identity is what we can synthetically call 'material identity'. This is the identity so far described by the sciences of inanimate nature. Compared with direct identity, the identities of presentation are distinguished by their multiplicity. The hypothesis put forward here is that psychic phenomena are phenomena characterized by the presence of a family of identities of presentation. This means that psychic objects are multi-stratified objects which systematically merge into the potential.[9]

We may therefore state that psychic agents possess a teleological component. Figuratively, a psychic agent is what it shall become. In these cases, the identity of the whole is never currently given in its entirety. Identity is therefore unfolding and constructional.

Conversely, the traditional identity of the world of nature is of the type: *x* is what it was. This yields the idea of identity as permanence – a situation in which it is not difficult to discern Aristotle's definition of essence.

The last two sections have raised a couple of very taxing problems (i.e., the problem of multiple dynamics and the problem of multiple identities). Let us try to devise a way to deal with them.

7. The main difference between state-transition systems and process systems

The main difference between kinematic or state-transition systems and process systems derives from a different understanding of time. According to the former, time is a parameter; according to the latter, time means 'timing'. Let us consider the parameter component first.

By parameter is meant a structure used for ordering (imposing order on) some group of items. Boldly stated, from a parametric viewpoint, *time is order*. It is a sequence of states which succeed each another in some manner. What really matters is 'what comes next', or what single state s_{n+1} follows state s_n (or which states $s_{(n+1)_i}$ follow a given state s_n). Ultimately, the fundamental relationship is a relationship of input-output type. For every state-transition perspective, the beginning point and the endpoint(s) are relevant; and important too is specification of what states the system passes through.

Process systems, by contrast, unfold continuously in time. They *are* time. For every processual perspective the important questions are: how fast is the system changing? how does the system unfold in time? how much time does the system spend in the vicinity of any given configuration (the concept of 'configuration' in the processualistic framework is meant as corresponding to the concept of 'state' in the state-transition framework)?[10]

The following questions are therefore inappropriate for a state-transition system and appropriate for a processual one: In what configuration was the machine at time 1.5? How long was the machine in configuration 1? How long did it take for the machine to change from configuration 1 to configuration 2?[11]

8. The ontology of processes

Understanding the differing importance of time for state-transition systems and process systems is of crucial importance, if a thoroughly developed process semantics is to be eventually achieved. As said, for state-transition systems time is a parameter. In the case of processes, time is instead a constitutive dimension. Processes are not *in* time, they *are* time; they are intrinsically temporal phenomena.

The difference between *being in time* (for example as a series of states localized somewhere) and *being time* is elusive and difficult to pin

down, and not just for the obvious reason that all temporal phenomena are also in time.

The problem is subtler than this, however. It concerns the fact that the basic structure of time is different in the two cases. In perhaps rather simplistic terms, we can seek to highlight what from the point of mathematical idealization seems to be their fundamental difference. Time as an external parameter has at most the nature of the reals and is therefore composed of uncountable 'atoms' ordered in the usual way. By contrast, the temporal continuum for processes seems to be composed of 'elastic' units, or units endowed with some sort of limited 'duration'. To adopt the usual spatial translation of time, these are units which possess some sort of extension.

It is perhaps of interest to note that Leibniz based his infinitesimal calculus on ideas very similar to those just outlined. His notion of infinitesimal segment was precisely that of an extension that is not nil but nevertheless can be considered zero. Weierstrass's well-known 'rigorous' reformulation of the infinitesimals into the usual ε-δ jargon provided a translation apparently able to prevent the mental cramp produced by Leibniz's formulation.

The matter would be definitively settled if the conceptual results of synthetic differential geometry were not now available. Before resorting thereto, let me provide a gentle introduction to the basic mathematical understanding of processes.

9. The basic mathematical model

The world is a dynamic reality. Material, psychological and social phenomena are all, for the most part, dynamic phenomena.[12] Dynamic phenomena of either sort (as both state-transition and processual) are at the heart of natural sciences. To our good fortune, the mathematics of dynamic phenomena has been closely studied. Leaving aside for the moment the case of a material item moving in some concrete space, what deserves a closer look is the case of an item changing its qualities (properties). The general mathematical framework for modeling dynamic items – that is changing items – can be represented as a manifold $\mathbf{I} = \mathbf{Q} \times \mathbf{T}$, where **Q** (for 'quality') and **T** (for 'time') are two submanifolds of **I** (for 'item'). At minimum, **Q** and **T** are two sets without any other structure. However, in order to obtain realistic models, it is necessary for them to have some structure (topological, differential, geometric).

Consider **T**, for example. This may be linearly or partially ordered, and if it is then to be measured, it will also have some metrics. Moreover, as the case may be, it will be continuous or discrete. The same applies to **Q**. More specifically, the structures of **Q** and **T** depend on the ontological category of **I**.

Manifold **Q** constitutes the space of variations, of degrees of freedom, of **I**: what is called the configuration-space of **I**. Given a certain item $i \in \mathbf{I}$, for every instant $t \in \mathbf{T}$, i will be in some position $q \in \mathbf{Q}$. The movements of **I** are the trajectories $m_{\mathbf{I}}$: $\mathbf{T} \rightarrow \mathbf{Q}$.

As usual, $\mathbf{Q}^{\mathbf{T}}$ denotes the mappings from **T** to **Q**. The elements of $\mathbf{Q}^{\mathbf{T}}$ can be transformed in various ways: for example, with transformations on **T** which do not alter the trajectories but the velocities of movement.

In general, a system depends not only on its configuration-space but also on its kinematics. This means selecting on $\mathbf{Q}^{\mathbf{T}}$ the set **K** of admissible kinematic movements. In this case, the characteristics of the elements of **K** depend on the characteristics of **Q** and of **T**. If **Q** and **T** are topological spaces, one would expect the elements of **K** to be continuous mappings from **T** to **Q**. If instead they are differentiable manifolds, the elements of **K** will be smooth of an appropriate order. Etc.

The eye of the philosopher sees at least two problems in the above structure. The first problem is that items are *only* seen as qualities in time. But qualities are qualities *of* something.[13] The information on the nature of the items under analysis provided by the markedly classical model that I have just mentioned are *only* given by the constraints imposed on **Q** and **T**. Nothing wrong here, obviously. But one may need a direct, explicit analysis of the items themselves.

Analysis of the link connecting the qualities to their bearers is lacking as well. And this too may be welcome.

10. Formal and ontological theories

A basic difference between formal (that is, mathematical, logic inclusive) theories and ontology concerns the way in which they consider the furniture of the world. From a formal viewpoint, the thesis may be maintained that universes of discourse are 'worlds of *points* (individuals)'. From an ontological viewpoint, the thesis should be maintained that universes are 'worlds of *particulars*'. Particulars, in their turn, may be subdivided into many different kinds of items. One possible list of basic particulars comprises objects, processes and stuffs and situations.[14] Each of

the latter categories requires its proper conceptual framework. This amounts to claiming that all of the above categories are endowed with a structure. In other words, none of them is a simple, unanalyzed atom. Some of the above categories are formally representable by individuals in a natural way (i.e., objects). For others (i.e., processes or stuffs) such representation is far less natural.

The next step is to study the connections and the dependences among the kinds of particular distinguished by one's preferred ontology. A traditional philosopher may try to reduce all particulars to objects (thus becoming a natural ally of the classically minded mathematician). A less biased philosopher will admit that *objects* may derive from *stuffs* plus (natural or artificial) *boundaries*; and that *objects* and *processes* may refer to each another. If she is of a processualist bent, she may claim that processes play a deeper (or wider) role, acting like the reference point of the field of particulars, or that all the particulars which are or appear different from processes are properly reducible to processes alone.

11. Synthetic differential geometry

The framework briefly described in section 6 may also be used to analyze the dynamics of items. In this case, items are representable as points moving in an underlying space. What really matters is that both cases (i.e., both the dynamics of qualities case and the dynamics of items one) treat time as a parameter, and the given dynamics as a state-transition between states. Moreover, there is no apparent way to connect qualities to their bearer and to distinguishing among ontologically different kinds of bearers (say, objects from processes). Synthetic differential geometry (SDG) may be a possibility.

The basic move of SDG is to change the concept of point. More precisely, SDG modifies the concept of infinitesimal quantity. According to SDG, an infinitesimal quantity can be taken to be a straight micro-segment "just long enough to have a slope … but too short to bend. It is thus an entity possessing (location and) *direction without magnitude*, intermediate in nature between a point and a Euclidean straight line".[15]

As far as time is concerned, Bell notes that it "can be regarded as a plurality of smoothly overlapping timelets each of which may be held to represent a 'now' (or 'specious present') and over which time is … still passing".[16]

Let us label ‘smooth’ the world as described by SDG. Among the results arising from its perspective, we should note the following:

1. In a smooth world any interval is indecomposable in the sense that it cannot be split in any way whatsoever into two disjoint nonempty parts;[17]
2. In a smooth world several distinct but possibly coincident proximity relations (algebraic, order-like and logical) can be distinguished.[18]

The new theory provides the conceptual background for development of a non-speculative, mathematically-based theory of *tendency* and *potentiality*.

Classically speaking, the life trajectory of actual items is characterized by the specific *direction* that it assumes at any one of its points and by the range of *possibilities* insisting on them. The matter becomes clearer if we consider respectively (i) the points of the trajectory as giving a vector in a state space and (ii) the set of points that can be arrived at from any specific point in one single step and for any specific set of global constraints insisting on the state space.

On the other hands, linelets are too small to have either possibilities or directions. Instead, they have potentiality and tendency, the latter being considered the micro-degree of the former.

Also to be noted is that the reformulation of infinitesimal calculus provided by SDG *does not alter* the results of the calculus. SDG clarifies the conceptual bases of calculus − and in doing so opens the way for new previously unforeseeable developments − but it does not modify the results of old-style calculations. One may care to remember that Copernican astronomy did not modify the results of Ptolemaic astronomy as well. The former only provided a better - ontologically more solid – framework than did the latter.

12. Intuitionism

The price to be paid for these results is the failure of the law of excluded middle.[19] In other terms, classical logic should be replaced by other kinds of logic. As Bell claims, the failure of the law of excluded middle “suggests that it was the unqualified acceptance of the correctness of this law, rather than any inherent logical flaw in the concept of infinitesimal

itself, which for so long prevented that concept from achieving mathematical respectability".[20]

The brief mathematical journey we have so far undertaken has therefore ended with a logically relevant result: *classical logic is unfit for a properly dynamical framework*. At best, it may be generalized so as to fit the needs of a state-transition theory.

We are still far from an adequately developed dynamic framework. In this sense, intuitionism is patently not the last word. In fact, intuitionisms may be seen as only a first step in a possibly very long journey. But, without intuitionism (and SDG), the journey does not start at all.[21]

13. Kinds of points

As said, SDG is a first step on a long journey. To foresee our destination, let me envisage some possible scenarios. A framework able to represent both state-transition and processual dynamics will obviously be highly desirable.

The elements of a state-transition system are static items. Their only admissible variation is the passage from one state to the subsequent one. Much more interesting is the case of SDG, where the items are, in a sense, elastic and display some kind of variation, according to some pattern. The obvious next step is the one in which even the pattern of variation exhibits degrees of variation.

State-transition dynamics comprise the lowest positive degree of (internal) variation. Let us call its items *thin* points,[22] which is meant points with a poor structure. An example may be of help. Consider the reals as modeled by a complete ordered field. Here, both order and algebraic structure are present, and the only automorphism is the identity automorphism.[23] This is an example of what I mean by 'poor structure'. The traditional response to this situation is to consider a different model which takes the reals to be an unbounded ordered set with denumerably dense subsets. In this case, the continuum only has the order structure, and there are obviously many automorphisms of this structure. In a sense, this is an example of a rich structure.

Unfortunately, this is not the kind of richness we are looking for, because it is still external to its points. What is required is a framework suitable for analysis of the points of a smooth world. As should be clear from § 7 above, accepting SDG is something like passing from thin points

to thick points, or in other words, passing from points with a poor category of automorphisms to ones with a richer category of automorphisms.

Suppose now that it is possible further to enrich the category of automorphisms insisting on the reals by adding new kinds of morphisms. By so doing we obtain a new category of reals which, for the time being, I shall call 'processual reals'.

If we adopt the phenomenologist's viewpoint, there must be at least three different kinds of automorphism insisting on processual reals. For convenience, we may call these morphisms of disposition, of memory and of anticipation. Dispositions concern the way in which a processual point reacts to incoming information. They pre-select the possible answers. Memories concern those morphisms which deal with the traces of the point's past history. Anticipations regard envisaged future achievements.

Suppose for instance a way is found to add automorphisms able formally to codify the variations undergone by the point. If so, it is possible to reconstruct the history of the point starting from the information embedded within the point itself. Put otherwise: by starting from the actually given point, it is possible to reconstruct those fragments of its past history that have left some kind of trace in the point itself.[24]

Some consequences follow:

1. The points of the processual continuum are thick. In other words, they have a significantly rich structure.
2. The new theory of infinitesimals referred to above may be considered an initial step in the direction I have envisaged.
3. From the presence of the above mentioned morphisms of memory, disposition and anticipation it follows that processual points depend on their 'life': different instances of processual points may have different stories.

The ideas sketched in previous sections can be taken as a phenomenologically based re-proposal of the Erlangen program. It may be known that the latter is now incorporated in a generalized form by the mathematical theory of categories.

Category theory becomes the natural framework for a process semantics theory. It may be recalled that the original ideas behind SDG can be traced back to Bill Lavwere, one of the founding fathers of category theory, and that its development is mainly due to categorically minded mathematicians. Moreover, category theory is the richest and most flexible known framework for integrating algebra and topology. Boldly: it is the

best known framework for putting logicians in touch with scholars well versed in any other mathematical field. Lastly, the internal logic of many interesting categories (toposes, among others) is intuitionistic logic. Classical logic may be validated by a topos only if particular conditions are assumed. In other words, the most general internal logic of toposes is the intuitionistic one. In still other words, classical logic has a mainly local nature.

14. Thick points are value-bearers

State-transition process semantics adopts the Galilean viewpoint (known as the Erlangen program in mathematics): knowing an item means knowing its answers. To know something, one has to place it in some context (often called experiment, or theory) and test it accordingly. The item's answers exhibit its nature. Boldly: the item *is* its answers.

Generally speaking, the above "let's try and see" methodology is fair enough. But there are cases in which it is flawed, if not utterly unacceptable (wrong).

Consider one extreme case. Any human being can be properly considered a social atom. Let us moreover assume that the dynamics of social atoms produce societies, from micro transient encounters (greetings, card games, etc) to the most structured ones (political and legal institutions, etc). Finally, let us assume that a social atom is defined by what it does. In most cases, this fits perfectly.

Now suppose that any of such atoms – after, say, a car accident – loses all its communicative capacities (no sight, no hearing, no speech, no movement). It is alive, but it cannot communicate. According to the "an item is its answers" methodology, it is no longer a social atom. From a pragmatistic viewpoint, this conclusion may be acceptable. I reject it.

Let us suppose that the above conclusion is unwelcome. What can we do? I am able to envision only two possibilities: either social atoms have an irreducible substance-like nature (the traditionalist viewpoint), or social atoms – like any other atoms – are thick. Otherwise stated, atoms have internal structure. The fact that an atom is unable to activate any communication channels does not imply that it is gone.

Leibniz may have been wrong in many respects, but his idea of monads as complex atoms is one of his enduring contributions to our understanding of reality. Even if his mathematics and his philosophy are thoroughly intertwined, the link has long been missing between the monadological viewpoint and the ideology of differential calculus, two of

his greatest theories. The concept of thick atom, endowed with tendency and potentiality, is possibly the first mathematically rigorous implementation of the idea of monad. From this it is a short step to the further conclusion that grown points are value-bearers. Therefore, they cannot be cancelled-out without harm.[25]

15. Conclusion

Process semantics is a demanding subject. It may be approached from different viewpoints.[26] The one chosen by Rescher in the Appendix to his *Process Metaphysics* has been the problem of non-existents. I have instead claimed that the problem of process semantics should be addressed from the point of view of the furniture of the world. After all, the underlying issue is the choice between object and process as the basic category of one's preferred ontology.

Moreover, I feel unable to share the analytic belief (apparently shared by Rescher) that logic is the conceptually most versatile and philosophically most fundamental tool. I would instead propose category theory as the main formal conceptual framework for understanding reality. The conceptual and formal problems briefly described in previous sections provide a few basic examples. Interactions between mathematics at large and logics may obviously prove fertile as well. I have tried to show that at least some of the basic issues of process semantics require ideas and tools arising from advanced contemporary mathematics. Insofar as formal codifications require a previous ontological understanding of the phenomena to be modeled, process philosophy may bring needed grist to the mathematical mill. But as far as contemporary philosophy is concerned, the philosopher should learn to look at what mathematicians have in store.

Notes

1. Rescher 1996, pp. 175-176.
2. For detailed treatment of the topic the obligatory reference is Cocchiarella 1986.
3. Cocchiarella 1986, p. 176.
4. Rescher 1996, p. 177.
5. Brentano 1981.
6. Useful surveys are Harel 1984 and Muskens, van Benthem and Visser 1997. The recent Harel, Kozen and Tiuryn 2000 provides a good introduction to the field from the viewpoint of computer science.
7. Marx, Polos and Masuch 1996, p. xi; Muskens, van Benthem and Visser, 1997, p. 620.
8. The case of the specious present is a striking example. Rescher 1996 notes in various passages that the specious present is a demanding problem for any processualistic viewpoint (well, for any viewpoint whatever).
9. Process-based philosophy shares with state-based philosophy a serious underestimation of the problem of levels of reality. For an analysis of the basic problems of the theory of levels see Poli 2001a.
10. The difference between state-transition and dynamic processes underlies Rescher 1996 as well. Unfortunately, Rescher veers frequently between the two interpretations and does not offer his reader an explicit analysis of their differences. In the context of branching time, the above difference may possibly (and partially) be traced back to the difference between "branching *in* time" and "branching *of* time" as presented in Rescher and Urquhart 1971, pp. 73-74.
11. van Gelder and Port 1995.
12. Material (physical, chemical, biological), psychological (presentative and representative) and social (educational, legal, business, sporting, etc) phenomena refer to different strata and layers of reality. A theory of the levels (strata plus their own layers) is required for understanding their many forms of connection. See the above cited Poli 2001a.
13. I agree with Rescher that *processes* can be unowned. On the other hands, qualities depend on their bearer, or arise from the interactions among bearers or between bearers and their environment. In the latter cases, they depend on processes, in the former they depend on something else and the nature of this latter item will depend on the accepted ontology (it may be a process, an object, a stuff or whatever particular is accepted by an ontology; see section 7). Anyway, processes play a substantial role.
14. In Poli 2001b, I have distinguished 8 different kinds of particulars: objects, processes, stuffs, disturbances, temporal and spatial groups, temporal and spatial mixtures (terminology slightly different from the one adopted here).
15. Bell 1998, p. 10.
16. Bell 1998, p. 10.
17. Bell 1998, 32.
18. Bell 1998, 93. Two more items of information are of general interest, namely that (a) the basic constructions of infinitesimal analysis can be cast in a form that is simpler and much more intuitive than the classical one, and that in a smooth world

decompositions like those underlying the Banach-Tarski paradox are provably false. For details see again Bell 1998.

19. The point was foreshadowed by Peirce as well: "if we are to accept the common idea of continuity … we must either say that a continuous line contains no points or … that the principle of excluded middle does not hold of these points. The principle of excluded middle applies only to an individual … but places being mere possibilities without actual existence are not individuals" (from a note written in 1903). Quoted by Bell 1998, p. 6.

20. Bell 1998, 15.

21. A further technical observation is relevant. It can be proved that the smooth continuum can be decomposed into two disjoint nonempty parts by removing a point from it. This is not true for the intuitionistic continuum, which is splittable only if at least a segment is taken off. This means that the smooth continuum is somehow intermediate between the classical and the intuitionistic continuum (cf. Bell 2001).

22. The 'thin-thick' terminology is borrowed from Albertazzi 2002.

23. 'Automorphism' and the subsequent 'category' are technical terms of the mathematical theory of categories. Very roughly, and without any mathematical detail, 'automorphism' may be understood as 'variation' of the items of a category (within the same category), and 'category' may be taken as 'theory'. The automorphisms insisting on an item themselves form a category. Macnamara and Reyes 1994 contains a basic introdution and a few cognitive-oriented applications of category theory.

24. For reconstruction of the past from the symmetries of the present see Leyton 1992.

25. Reference to grown or lived points is mandatory for avoiding the strictures of fundamentalists. The difference between ungrown and grown points may be grounded in the difference between potentiality and possibility, a difference absent from contemporary discussions. To make a long story short, possibilities are *constrained* potentialities. The former alone may constitute realities, in the following sense: realities are paths in a space of possibilities (see section 6 above). A states-transition understanding of processes may remain with possibilities. But possibilities are not enough for a thoroughly dynamic understanding of processes.

26. Among other things, proper awareness of the develoment of process philosophy may provide better understanding of the theory. In this regard, the ample conceptual and historical reconstruction offered by Rescher displays an interesting cognitive blindness. It is obviously true that process philosophy has been a significant component of American philosophy (as widely proved by the names, among many others, of Peirce, Whitehead, Pepper, Ushenko and Hartshorne). But it is equally true that process philosophy (granted, under different denominations) has been a significant component of European philosophy as well, as proved by the names of Brentano, Husserl, and Hartmann (among many others). Incomprehensibly, none of them is ever quoted in Rescher's otherwise stimulating book.

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