DESCRIPTIVE, FORMAL AND FORMALIZED ONTOLOGIES

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1. OVERTURE[1]

I shall distinguish descriptive, formal and formalized ontology. Each of these ontologies comes in two guises: domain-dependent and domain-independent. Domain-dependent ontologies concern categorically closed regions of being; on the other hand, a domain-independent ontology may be properly called general ontology.

Ontologies should have a sound methodological basis. Adequate ontologies are those which satisfy the following requirement: what is predicated must be true of the class of items with respect to which it is predicated.

> Many theories [= ontologies], comprising no fallacy, are yet inadequate: one may form the concept of "a cigar weighing five ounces", predicate about that class everything known about material things in general (about solid bodies in general, the chemical properties of the ingredients of these cigars, the influence of smoking them on health, and so on); these "theories" – while perfectly correct – are manifestly inadequate since what is predicated with respect to "cigars weighing five ounces" is also true of innumerable objects which do not belong to that class, such as cigars in general.

1. Thanks are due to R. Corazzon, L. Ekeberg and B. Smith for their comments to an earlier version of this paper.

D. Fisette 2003. *Husserl's Logical Investigations Reconsidered*, pp. 183-210.

> A theory may be inadequate either (1) because the predicates are related to classes which are too narrow ... or (2) because the predicate is related to a class which is too broad (such as various sociological theories which attribute "everything" to the influence of one factor which in fact plays a much more modest part) (Petrazycki, 1955, p. 19).

Petrazycki developed the above methodological perspective in 1905, in St Petersburg. His viewpoint comes very close to the shortly previous perspective developed by Husserl in Halle, and in particular to the idea of organic theory or of *Mannigfaltigkeit* (as for the latter, cfr. Hill, 2000).

Petrazycki's conditions may be more clearly stated by claiming that adequate theories must be (i) universal, and (ii) closed. Closed universal theories may be aptly called natural theories.

Following on the path opened by such thinkers as Husserl, Hartmann, Peirce and Whitehead (and first of all by Aristotle), in this paper I shall adopt a categorical viewpoint. Resorting to a categorical viewpoint means looking for "what is universal" (either in general or in some specific domain). Those with a grounding in contemporary mathematics will recognize here the similar claim advanced by Bill Lawvere some decades ago: category theory, as a foundational theory, is based on "what is universal in mathematics" (1969, p. 281).

2. DESCRIPTIVE, FORMAL AND FORMALIZED ONTOLOGIES

The unity and the variety of the world is the outcome of the complex interweaving of dependence connections and forms of independence among the many items of which it is composed. I shall seek to explain the features of this multiplicity by beginning with an apparently trivial question: what is there in the world?

We may say that there are material things, plants and animals, as well as the products of the talents and activities of animals and humans in the world. This first almost trivial list already indicates that the world comprises not only things, animate or inanimate, but also activities and processes and the products that derive from them.

It is likewise difficult to deny that there are thoughts, sensations and decisions, and the entire spectrum of mental activities. Just as one is compelled to admit that there are laws and rules, languages, societies and customs.

We can set about organizing this list of objects by saying that there are *independent items* that may be *real* (mountains, flowers, animals, and tables), or *ideal* (sets, propositions, values), and *dependent items* which in turn may be *real* (colours, kisses, handshakes and falls) or *ideal* (formal properties and relations).

All these are in various respects items of the world. Some of them are actually exemplified in the world in which we live; others have been exemplified in the past; and yet others will possibly be exemplified in the future (Poli 2001, ch. 5).

Descriptive ontology concerns the collection of such *prima facie* information either in some specific domain of analysis or in general.

Formal ontology distills, filters, codifies and organizes the results of descriptive ontology (in either its local or global setting). According to this interpretation, formal ontology is formal in the sense used by Husserl in his *Logical Investigations*. Being "formal" in such a sense therefore means dealing with categories like *thing*, *process*, *matter*, *whole*, *part*, *and number*. These are pure categories that characterize aspects or types of reality and still *have nothing to do with the use of any specific formalism*.

Formal codification in the strict sense is undertaken at the third level of theory construction: namely that of formalized ontology. The task here is to find the proper formal codification for the constructs descriptively acquired and formally purified in the way just indicated. The level of formalized constructions also relates to evaluation of the adequacy (expressive, computational, cognitive) of the various formalisms, and to the problem of their reciprocal translations.

The close similarity between the terms "formal" and "formalized" is somewhat unfortunate. One way to avoid the clash is to use "categorical" instead of "formal".

Most contemporary theory recognizes only two levels of work and often merges the level of the formal categories either with that of descriptive or with that of formalized analysis. As a consequence, the specific relevance of categorical analyses is too often neglected.

The three levels of ontology are different but not separate. In many respects they affect each other. Descriptive findings may bear on formal categories; formalized outcomes may bear on their twin levels, etc. To set out the differences and the connections between the various ontological facets precisely is a most delicate task.

3. VARIANTS OF FORMALIZED ONTOLOGY

Formalized ontology presents two main variants, depending on the preferred formal environment. Its mainstream acception lies within a *logical* version of formal ontology; the other version being characterized by the use of other mathematical environments (algebraic and/or geometrical). In its turn, the logical interpretations of formal ontology can be further subdivided between those working with classical or otherwise 1st order predicate logic and those working with 2nd order logic.

Generally speaking, philosophers seem less acquainted with non-logical ontologies, which are much more widespread in science, and I shall therefore confine myself to a logical framework.

4. A FEW DATA ABOUT FORMAL ONTOLOGISTS

A small dictionary of philosophers who have *explicitly* dealt with formal ontology would be useful. Two observations are important: (1) in this section the expression "formal ontology" will be used in the broad sense to refer to both the formal ontology and the formalized ontology described in the previous section; (2) the qualification "explicitly" is crucial. In effect, the range of formal ontology (in the sense given sub (1) above) is so broad and so ramified that it is difficult to say who has not dealt with it. But if we employ as our criterion the use of the expression "formal ontology" (or something similar) in a sense consistent with the one specified, we find that the list of authors diminishes considerably.

The point of departure is obviously Husserl's *Logical Investigations*. The author who more than anyone else has developed the categorial analysis of ontology is Nicolai Hartmann. As regards phenomenologists, the Husserlian who has paid closest attention to the theme is Roman Ingarden, especially in his monumental work D*er Streit um die Existenz der Welt*. Formal domain ontologies have been developed by Ingarden himself (the domain of artistic phenomena with particular regard to literary works and the domain of values), Hartmann (natural world, social world, art, values), Scheler (values), Reinach (law), Stein (the concept of person), and Plessner (the social world).

Among analytic philosophers, we find a constant interest in the relationships between the dimensions of the formal and of ontology from Carnap onwards. Authors who warrant at least brief mention are certainly Goodman, Prior and Quine. More difficult to classify for various reasons are the theories of Bunge and Sommers. Johansson has developed an innovative categorical approach which reveals the influence of the Brentanian tradition (Husserl and Marty in particular) as well as the Marxian tradition, especially in his analysis of social action.

Nino Cocchiarella, Kit Fine and Jerzy Perzanowski are perhaps the most notable of philosophers currently conducting explicitly formal analysis. Cocchiarella has worked in particular on problems of predication and nominalization (issues explicitly analyzed by Husserl), systematically reconstructing so-called theories of universals (nominalism, conceptualism and realism, the latter two with important variants) in a formally homogeneous environment. Of Fine's many works, particular mention should be made of those which formally reconstruct various fundamental concepts of the philosophical tradition (the concept of substance among others), often starting from their Aristotelian bases. Perzanowski has developed an innovative account of ontology within a Leibnizian framework. From a formal point of view, a distinctive feature of his position is the idea that there are formal structures which precede the distinction between the propositional and the predicative levels and require particular algebraic codification (Perzanowski, 1996). One aspect to be noted is that all three of these philosophers work in explicitly formal terms while simultaneously paying close attention to Husserlian matters (Cocchiarella has analysed the already-mentioned problems of predication and nominalization; Fine has developed a sophisticated algebraic reconstruction of the third *Logical Investigation*; Perzanowski was one of Ingarden's pupils).

In the past twenty years, a group of mainly (but not exclusively) analytic philosophers have drawn on the work of one of Brentano's pupils to develop new formal tools. I am obviously referring to so-called Meinongian semantics, the history of which divides into two main periods. The first was during the mid-1980s and is particularly closely associated with Lambert, Parsons, Rapaport, Sylvan and Zalta. These are authors whose names establish further connections with free logics, relevant logics and paraconsistent logics. The second, more recent, period is associated especially with the names of Jacquette and Pasniczek (works reviewed in Poli, 1998b and 1999).

One author who has engaged in dialogue with those just mentioned, although he developed his own and original point of view, was Hector-Neri Castañeda, whose guise theory proposes a wide series of predicative structures both ontological and cognitive. Castañeda's premature death prevented further development of his theory and it remains incomplete.

Also to be mentioned is a minor, mainly American philosophical tradition which although it lies outside the analytic tradition has nevertheless made a major contribution to formal ontology. I refer to the tradition of "dynamic ontology" developed by Peirce, Whitehead, Butler and Hartshorne and which a fine book by Rescher has recently revitalized (cfr. Rescher, 1996). Also linked with this tradition is the interesting school of "process theology".

Despite its apparent diversity, the "dynamic" tradition in the English-speaking countries has taken up positions which come significantly close to those developed by the German-speaking sister tradition associated with the names of Brentano, Husserl, Meinong and Hartmann. Thorough comparison between the two traditions has yet to be made (worth mentioning among the few that I know is Mohanty, 1957).

Other areas of inquiry are Perry and Barwise's situation semantics and Suszko's non-Fregean logics. While the work of the former two authors is so well known that it requires no introduction, Suszko's deserves closer analysis. This I shall provide below when discussing the problem of the identity connective.

Lying midway between the analytic and phenomenological traditions are the studies of Barry Smith and Peter Simons, who deal in particular with the theory of parts and the development of a general mereology which, according to Smith, constitutes the fundamental instrument of ontology.

Studies which find inspiration in phenomenology and draw their tools from algebraic topology has been developed by Jean Petitot, who studied under René Thom and has continued his catastrophe theory.

Finally, my own work seeks to overcome the limitations of the two schools of dynamic philosophy (the German "camp" of Brentano and his followers, and the American "camp" of Peirce and Whitehead) by developing a dynamic theory of substances which comprises various interacting sub-theories, principally those of particulars, of the levels of reality, and of wholes (Poli, 2001).

These, therefore, are names of the philosophers currently at the forefront of work in ontology.

Before closing this section, I would point out that the term and idea of "ontology" have begun to enjoy currency in various sectors of artificial intelligence, and particularly in (i) the representation of knowledge; (ii) theory of databases; (iii) natural language processing; and (iv) automatic translation. In short, those who most frequently talk about ontology are researchers in the acquisition, integration, sharing and re-utilization of knowledge. Ontology comes into play as a viable strategy with which, for example, to construct robust domain models. An ontologically grounded knowledge of the objects of the domain should make their codification simpler, more transparent and more natural. Indeed, ontology can give greater robustness to models by furnishing criteria and categories with which to organize and construct them; and it is also able to provide contexts in which different models can be embedded and re-categorized to acquire greater reciprocal transparency (Poli, 1996; Poli and Mazzola, 2000; Poli, 2001).

5. A NOTE ON HUSSERL'S CONCEPTION OF FORMAL ONTOLOGY

I have already mentioned that the concept of formal ontology was first formulated in Husserl's *Logical Investigations*. Perhaps the most significant aspect, however, is that although the concept was subsequently refined and articulated, its essential features remained substantially unchanged through the many subsequent ramifications of Husserlian theory. This fact should be emphasized because it shows that formal ontology has proceeded independently of the more strictly phenomenological developments of Husserl. My evidence for this assertion consists in the fact that formal ontology deals with the object as something that exists, that has already been constituted, and that is somehow given. It does not concern itself with the process (phenomenological or metaphysical) by which the object is constituted, nor with its modes of subjective givenness (Poli, 1993).

For Husserl, a prime distinction to be drawn is between formal and material concepts. Material concepts are typically exemplified by nouns, whether these are proper, general or collective, or whether they are concrete or abstract. The question of formal concepts is less straightforward because it is necessary to distinguish between the domain of the logically formal and that of the ontologically formal. The former comprises logical operators and functors, the latter concerns whatever pertains to the "object in general" or the "simple something" (*Third Logical Investigation*). Logical formal concepts are thus negation, conjunction, implication and quantifiers. Ontological formal concepts are: object, state of affairs, unity, plurality, number, relation, connection (*Prolegomena*), subject and determination, individual, species and genus, quality, cardinal number, order, ordinal number, whole, part, magnitude (*Third Logical Investigation*).

On completion of the *Logical Investigations*, Husserl turned his attention to mainly genetic inquiry into the inner consciousness of time, returning to descriptive (noematic) analysis in *Ideas I*. The formal sciences now comprised "the formal-ontological discipline represented by, besides formal logic … the other disciplines of *mathesis universalis* (among which arithmetic, pure analysis, the doctrine of variety)". *Ideas I* goes further than *the Logical Investigations* by distinguishing the ontological categories into syntactic categories and substrate categories. The former are derived forms of objectuality and comprise: states of affairs, relation, unity, multiplicity, number, order, ordinal number, etc. Removing the syntactic form from the syntactic categories yields objects that are no longer syntactic-categorical formations, so that we have as yet unformed correlates of the functions of attributing, denying, connecting, enumerating, etc. The ultimate categories of the substrate finally divide into the two categories of "ultimate material essence" and pure individual singularity or "this here and now".

Once essences have acquired form they become part of a hierarchy of species and genera. The specific and general relationships among essences are not "relationships of classes or sets" because they are intensionally characterized, given that a more general essence is "contained in immediate or mediate fashion" within a singular one, in a *whole/part relationship*. Parts are therefore to be understood as moments (i.e. as distinguishable but not separable parts).

Like all objects also essences have content and form (Appendix 1). The content of the objects of formal ontology is not a material content connected with the categories of some material region. Rather, it is a content relative to the pure form of region and it is obtained from the idea of objectuality in general, perhaps categorically transformed by nominalizations. These furnish formal ontology its concepts by performing formal categorical transformations of the idea of objectuality in general.

To summarize: emphasized in the passage from *Logical Investigations* to *Ideas I* are four main aspects which find further development in *Formal and Transcendental Logic* and *Experience and Judgement*. These problems pertain to the following: the doctrine of the forms of logical meanings as a sub-level of a scientific *mathesis universalis*; the distinction between syntactic form and ultimate substrate; nominalization and use of the theory of parts and the whole in analysis of the relationships among essences.

In *Formal and Transcendental Logic*, Husserl reiterates the "inseparable unity" of formal logical and mathematics "in the idea of a formal *mathesis universalis*", adding that "the authentic sense of a formal ontology" resides in the pure analytic of non-contradiction which concerns "everything possible and everything thinkable".

Further ramifications of the theory are to be found in *Experience and Judgement*. The distinction between syntactic forms and substrates is articulated into the further distinction between syntactic or categorical forms and nuclear forms. Thus every predicative judgement is characterized by a twofold "putting-into-form": a nuclear putting-into-form as substantivity and adjectivity, and a syntactic putting-into-form as subject, predicate, etc. The logical subject and predicate have respectively the nuclear form of substantivity and adjectivity, where the former designates the being-for-itself or the substantiality of the object, while the latter designates the being-in-something-else or the insubstantiality of the object. In Fregean terms, this is the difference between saturatedness (of the subject) and non-saturatedness (of the predicate).

Note also that we can always use nominalization to substantify an insubstantial object. If we apply this procedure to the substrate/determination distinction, we find that a substrate may be the substantialization of a property, as in the classic shifts from "white" to "whiteness" or from "beautiful" to "beauty". However, if we eliminate the intervention of nominalization, we must eventually arrive at the absolute substrates and, likewise, at the absolute properties. As regards formal ontology, which is our sole interest here, the only pertinent aspect of the absolute substrate is the one founded on something that is logically entirely indeterminate. The subsequent articulations of this substrate divide between substantial (pieces) and insubstantial (moments). This immediately raises the question of the division of the object, which according to Husserl can be distinguished into pieces and moments. The former divide into substantial and insubstantial pieces. Substantial pieces are the parts of the object that are external to each other; insubstantial pieces are instead parts that co-penetrate each other.

Moments divide between immediate and mediate. Immediate moments are the properties of the insubstantial pieces and the forms of connection among substantial pieces. Mediate moments differ from the properties of the object in that they are properties of the moments of the object.

In short, for Husserl formal ontology is characterized by the presence of the following sub-theories:

- theory of the nuclear forms and syntactic forms;
- theory of parts and the whole;
- theory of nominalization.

I shall not discuss nuclear and syntactic forms any further here. Instead I shall now deal briefly with the theory of parts and the problem of nominalization.

6. MEREOLOGIES

To be precise, the theory of parts and the whole concerns both formal ontology and material ontology. It pertains to the former as the pure theory of independence and non-independence, and to the latter as regards the particular laws of non-independence which apply in the various ontological regions.

Mereologies (or theories of parts) are generally classified into extensional and intensional. The former are ontologically monistic: every object that exists is an object; the parts of objects are objects; and compositions of objects are objects as well. This is the position put forward by Lesniewski, who starts from his interpretation of ontology where he gives formal definition to the concept of "object" and then extends it in his mereology based on the notion of being a "proper part of", then developing it further in his theory of time and space. Intensional mereologies by contrast distinguish the parts of an entity into independent and non-independent parts. The former are termed "pieces", and they are those that effectively assume the denomination "part", while the latter are called "moments". From an ontological point of view, parts are objects in the same sense that the entities of which they are parts are objects. Moments have a different ontological valence. They are secondary objectualities and solely in the translated and subjective sense. Which, however, does not mean arbitrary. Note the formulation employed by Husserl, who speaks of the independence of parts and the non-independence of moments, not of independence and dependence. The difference is a subtle one but it is deliberately introduced. The reason for it resides in Husserl's mathematical training, where the use of negation in these cases signifies that equality is possible. When *a* is said to be not-greater than *b*, this means that *a* is less than *b* or that *a* is equal to *b*. Translated into our present case, when one states that *a* is non-independent of *b*, the intention is to say that *a* is dependent on *b* or that *a* is equal to *b*. Moments may therefore be equal to the whole of which they are moments, where, however, the concept of equality should be understood in the sense of indiscernibility. The possible indiscernibility of the moments from the whole should not be confused with the possible identity of the part (as distinct from the proper part) with the whole. A part may even be the whole itself, while a moment can at most *coincide* with the whole; or in other words, it may be indiscernible from the whole but it is nonetheless distinct from it.

7. NOMINALIZATION

Let us consider the standard situation of a statement taking the form P(*a*), where *a* is the subject and P is the predicate. The subject denotes an object, i.e. something that is independently saturated, ontologically self-sufficient and complete, and which does not need to be determined further. The predicate is instead something that is intrinsically unsaturated and which requires the noun to acquire completeness. Predicates correspond to concepts. By means of nominalization we obtain a situation of the type F(P), where the original predicate appears in the guise of a noun in the subjective position. In this case, P is no longer unsaturated as it was in P(*a*), but it has the same subjective characteristics as *a*, except that corresponding denotatively to *a* is an entity in the universe of discourse. May we therefore say that also corresponding to the predicate P is an object of the same type as the one that corresponds to *a*? Obviously not. Frege says that corresponding to a nominalized predicate is a *conceptual correlate* which has individual value and is therefore saturated. Between the predicate and the nominalization of the predicate, i.e. between "P" and "the concept P" to use Frege's expression, there is a relationship of *representation*. That is to say, in this situation "the concept P" becomes the *individual* representative of "P", where the latter stands as the argument and the former as the value of the representation function.

Objects denoted by nominalized predicates are *intensional entities*, or in other words, properties and relations which have their own abstract form of individuality. We thus find ourselves in a situation where there are objects and conceptual correlates endowed with their own specific individuality, as opposed to a lack of individuality by concepts. A concept as such, in that it lacks individual characteristics, cannot be part of any universe of discourse.

With the nominalization of *P* in the sense of "the concept *P*" one obtains an individual term in the theory of concepts which denotes not concepts but special objects. For detailed treatment of the topic the obligatory reference is Cocchiarella 1986.

From a formal point of view, the combination of the various cases can be shown as follows Cocchiarella (1986, ch. 4):

(Abelard =)	$[\forall F^n][\neg\exists x](F = x)$
(Abelard ≡)	$[\forall F^n][\neg\exists x](F \equiv x)$
(Plato =)	$[\forall F^n][\exists x](F = x)$
(Plato ≡)	$[\forall F^n][\exists x](F \equiv x)$

where "≡" is used as the sign for indiscernibility, namely $a \equiv b =_{df} \forall F[F(a) \leftrightarrow F(b)]$.

Abelard claims that nominalized predicates are singular terms that fail to refer (to any *thing*). The connections between the various theses above are as follows:

- (Abelard ≡) implies (Abelard =), that is to say: *indiscernibility implies identity*;
- (Plato =) implies (Plato ≡), that is to say: *identity implies indiscernibility*.

It is worth noting that Russell's antinomy requires (Plato =). Applied to $\exists F\forall x(F(x) \leftrightarrow \exists G[x = G \wedge \neg G(x)]$, Russell's argument shows that the assumed concept is a non-thing, i.e., $\forall x(F(x) \leftrightarrow \exists G[x = G \wedge \neg G(x)]) \rightarrow \neg\exists x(F = x)$ (Cocchiarella 1986, p. 176).

8. THE FORMS OF REPRESENTATION

An interesting articulation of the point of view being presented here is the idea that the saturation of the concepts forming atomic statements is not, as Frege maintained, a truth value but rather a *mental act* or a *speech act*. Non-saturation thus consists in the purely *dispositional* state of cognitive capacities. The *exercise* of these capacities informs the mental *acts* of a predicative or referential nature. A categorical judgement thus becomes a mental act consisting in the combined application and mutual saturation of a referential concept and a predicative concept.

The non-saturatedness of the concepts also means that they may stand as logical subjects by means of their individual representations. An aspect that has perhaps not been sufficiently emphasized is that there are numerous modes of saturation. In general, there will be as many modes of saturation as there are modes of predicating the concept. From a logical point of view, the range of these modes comprises at least the following cases:

- *copula*: associates the concept with an object as its individual bearer (concept as property)
- *logical choice*: associates the concept with its official representative, as in Hilbert's theory of ε-terms (concept as one in particular);
- *generalization*: associates the concept with its general representative (concept as one in general);
- *denomination*: associates the concept with the linguistic representative that expresses it (concept as *vox*) (Bernini, 1987).

9. THE LOGICAL AND THE ONTOLOGICAL

We have seen that ontology comes in various guises. Logic itself presents a number of different facets. In this regard, the following quotation from Gödel is exemplary in its clarity:

> Mathematical logic … has two quite different aspects. On the one hand, it is a section of Mathematics treating of classes, relations, combinations of symbols, etc. instead of numbers, functions, geometric figures, etc. On the other hand, it is a science prior to all others, which contains the ideas and principles underlying all sciences (Gödel, 1944, p. 123).

Gödel's description of mathematical logic as the science that precedes all others is entirely similar to the definition of metaphysics provided by Aristotle. It is accordingly natural to think that mathematical logic (in Gödel's second sense) is able to depict the deepest-lying structural maps of reality. The two aspects of mathematical logic distinguished by Gödel can be respectively denoted with the expressions, taken from Heijenort, 1967, of "logic as calculus" and "logic as language" (cfr. Cocchiarella, 1974 and 1988).

Their difference resides in that logical form can be used to represent many different things: logical validity, truth conditions, an abstract calculus, cognitive aspects or even *ontological structures*. Gödel's reading suggests that logic (as language) is formal ontology.

The thesis of the identity of logic (as language) with (categorical) ontology is, as we know, one of the essential assumptions of Platonism (or logical realism). Moreover, it is one of the fundamental theses of the Hegelian dialectic and its paralogisms.

Unfortunately, the connections between the ontological and formal levels are much more complicated than would appear from Godel's observation. I shall therefore resist Gödel's solicitations and refrain from identifying logic (as language) *sic e simpliciter* with ontology.

Let us assume that categorical ontology and the formal sciences both have the task of searching for what is universal, in general or in some domain. The problem is that we have no reason to think that what is universal in the formal sense must perforce coincide with what is universal in the ontological sense. In other words, it is advisable to keep what is universal in the formal sense separate from what is universal in the ontological sense (unless there is some strong argument to the contrary).

One way to view the difference between the two accounts of universality is as follows. Let us imagine that the structures of universality concretize themselves in simplicity, in accordance with the ancient motto *simplex sigillum veri* ("simplicity is the seal of truth"), so that the more universal structures are also simpler (according to some criterion of simplicity). If this were true, ontologically simpler (and therefore more universal) structures should also be formally simpler (and vice versa).

Perhaps the most elementary example is the one that follows. Consider the universe of states of affairs and the propositions that refer to them. As we know, when adequately rigid ontological conditions are imposed upon states of affairs (i.e. upon the objects of the universe of discourse), the usual propositional connectives are interdefinable. If we instead relax the ontological conditions and therefore characterize the universe of discourse in a more universal way, connectives are no longer interdefinable (as in intuitionist logic).

Greater ontological rigidity may therefore simplify the formal level, but greater ontological universality may make it more complex.

10. DUALITY

In the light of these considerations we may assume that it is possible to distinguish logic (as language) from ontology. Having said this, however, perhaps the most interesting aspect of the relationships between logic and ontology concerns neither their unification nor their distinction but their *correlation*, or in other words, their reciprocal influence. An area of particular significance for the study of the correlations between logic and ontology is that of duality phenomena (in both the formal sciences and ontology).

Duality phenomena are well known in the formal sciences: suffice it to mention situations like ideal / filter, open set / closed set, join / meet, intersection / union, universal quantifier / particular quantifier, it-is-necessary-that / it-is-possible-that. While analyzing the numerous cases of formal duality, Rosado Haddock (1991) points out that (1) the feature shared by duals is their interderivability within a particular theory, and that (2) interderivable propositions can be considered to be different states of affairs which relate to the same situation of affairs. Rosado Haddock thus takes up Husserl's distinction between *Sachverhalt* and *Sachlage*. David W. Smith (1991) objects to the strictly formal criterion used by Rosado Haddock on the ground that Husserl's distinction between states of affairs and situations of affairs is not essentially formal but mainly ontological.

Strictly speaking, however, both Rosado Haddock and Smith are right, because duality is a phenomenon with both formal and ontological aspects. In what follows we shall also see that the discussion between Rosado Haddock and Smith has failed to address what is perhaps the most interesting aspect of duality: the presence of two sharply distinct families of dual phenomena.

11. STATES OF AFFAIRS AND SITUATIONS OF AFFAIRS

The following example is taken from Aristotle and was quoted by Anscombe and Geach, 1961:

> The road that leads from Athens to Thebes is the same road that leads from Thebes to Athens: but in the former case it goes uphill, while in the latter it goes downhill.

We may therefore say that this road, *qua* road from Athens to Thebes, is uphill, and that this same road, *qua* road from Thebes to Athens, is downhill. One thus concludes that there is a relation between the object and the standpoint or the point of view of looking at it. In entirely similar terms, Anscombe states that, "there aren't such objects as an A *qua* B, though an A may, *qua* B, receive such-and-such a salary and, *qua* C, such-and-such a salary" (Anscombe 1981b, 208; the role and the properties of the functor *qua* are analyzed in Poli,1994 and 1998a).

It is well known that, for Husserl, cases like "A is part of B", "B contains A as its part", are two *states of affairs*, which have the same *situation of affairs* as their foundation. Corresponding to the same situation of affairs may be two or more states of affairs; in the same way as corresponding to the same state of affairs may be two or more propositions (thoughts) (Rosado Haddock 1991).

There follow examples of states of affairs that refer to the same situation of affairs (Smith, 1991):

- Heidi is Hans's wife; Hans is Heidi's husband;
- The glass is half full; the glass is half empty.

Husserl constructs states of affairs from situations of affairs on the basis of the following principles:

- States of affairs are categorical structures;
- Situations of affairs are precategorial entities;
- States of affairs are founded upon situations of affairs;
- Different states of affairs may be founded upon the same situation (Smith 1991, p. 52).

While commenting on these principles, David W. Smith adds:

> Unfortunately, the principles Husserl put forth do not tell us what situations are. Indeed, some four different intuitions might lie behind these principles, yielding four different notions of situation, outlined as follows:
>
> 1. […] Situations are just *species*, or types, of states of affairs […]
> 2. States of affairs might be thought of as *aspects* of situations (in effect reversing the first proposal) […]
> 3. A situation might be viewed as the matter (*Sache*) from which different states of affairs are formed […]
> 4. A situation might be viewed as a certain sort of *part-whole* complex, or affair (*Sache*), from which parts are extracted and put together into states of affairs […]
>
> What remains unclear, then, is the kind of unification that form a situation" (Smith 1991, pp. 53-54).

The following table summarizes the situation:

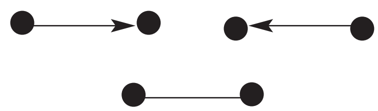
Table 1.

Situation of affairs	State of affairs
Species	Instance
Instance	Aspect
Matter	Form
Whole	Part

In reality none of these oppositions seems to be correct. Consider the difference between "The road that leads from Athens to Thebes" and "The road that leads from Thebes to Athens". Their difference seems manageable between descriptions based on "pure" or "a-directed" relations as opposed to descriptions based on "directed" relations. Consider the difference between the following directed and a-directed diagrams:

The two directed graphs in the first row can be interpreted as "the road that leads from Thebes to Athens" and "the road that leads from Athens to Thebes". These interpretations are explicitly linked to a direction. On the contrary, the graph drawn below "says" *only* that the two vertices (points) are connected. In other words, it exemplifies only the pure situation of "being related to". It is clear that the graphs in the first row result from the graph drawn in the one below as soon as an indication of direction is added (and vice versa).

We may therefore distinguish between two representational spaces: a space composed of situations of affairs (a-directed graphs), and a space composed of states of affairs (directed graphs).



If this reading of the difference between states of affairs and situations of affairs can be generalized, then the relationship between situations and states would have the nature of a theory-theory connection and would not correspond to any of the four oppositions in Table 1.

12. UNFOLDED STATE OF AFFAIRS

The situation can be further elaborated by introducing a third representational space that mirrors the difference between the ontological viewpoint and the judgements. Daubert first discussed this opposition. In the words of Karl Schuhmann and Barry Smith:

> To capture this opposition between how things stand in themselves and how they are asserted to be in our judgements, Daubert [...] distinguishes between the *Sachverhalt* and what he calls "*Erkenntnisverhalt*", the "state of affairs as cognized" or as "unfolded" in cognition [17v]. The former is the objectively existing structure of things, properties and relations as they are in and of themselves. The latter is that side or aspects of the former which serves as the immediate objectual correlate of a given concrete act of judging [...]
> Thus consider: "The chairman opens the meeting", "The chairman is opening the meeting", "The meeting is being opened by the chairman", "The opening of the meeting is being conducted by the chairman", "The chairman has opened the meeting", "The meeting has been opened by the chairman" [17v, 63r]. Each of these sentences differs as to its associated state of affairs as cognized, but they are in fact concerned with one and the same objective *Sachverhalt* (Schuhmann and Smith, 1987a, pp. 367-8).

The examples quoted require a theory able to unify them into a coherent whole. This will contain "chairman", "meeting", "opening the meeting", and so forth, as primitive or derived terms. It will be about the worldly structure of meetings—that is, it will be about how things are.

Table 2.

Ontological	Logical
Erkenntnisverhalt	*Utterance*
Sachverhalt	*Sentence*
Sachlage	*Situation*
(Evidence)	*Truth-value*

It follows that at least three different levels can be distinguished: the level of the *Sachlage*, the level of the *Sachverhalt*, and the level of the *Erkenntnisverhalt*. From a formal point of view, aspects of the latter can be represented by allowing more edges between two adjacent vertices (as in multigraphs).

Let us now try to construe the logical table corresponding to the ontologically-grounded distinctions so far considered.

Both sides present a many-one form of connection. As regards the logical side, this means that many utterances may express one sentence; many sentences may express one situation; and many situations may express one truth-value. By resorting to other conceptual frameworks, we may also say that utterances instantiate judgments; sentences instantiate senses (categorical objectualities); situations instantiate things-in-themselves (pre-categorical objectualities); and truth-values instantiate … well, nothing but themselves.

In their turn, all of these are values of a common, basic argument. Call it "proposition". We will say that propositions have utterances, sentences, situations and truth-values as their functional values. What this means will soon become clearer (see § 15 below).

The ontological side presents the very same many/one structure as the logical one. Many unfolded states of affairs may refer to one state of affairs, etc. The ontological rock-bottom level represented by "evidence" has been bracketed because used in this way it is a more Brentanian than Husserlian concept. All the ontological categories are values of a common argument which may be called "thought" or "content".

Let me summarize the discussion thus far. On analysing a number of classic situations of categorial duality, we have seen the possibility of an underlying precategorial unification (the relationship between states of affairs and situations of affairs). The unification therefore acts as a quotient or invariant. I then generalized the procedure by unifying a multiplicity of unfolded states of affairs in their underlying state of affairs. The two steps were made uniform by constructing a table of quotients in which multiplicities of items were progressively unified at increasingly profound levels. The table of quotients was also constructed in two different guises, one formal and one ontological.

If we operate in this manner, duality phenomena become a particular case of a more widespread phenomenon. On this basis, which we may take as given, we can now deepen our examination of categorial dualities.

13. ONTOLOGICAL DUALITY

The importance of duality emerges as soon as one understands that there are two significantly different families of dual phenomena distinguished by the result obtained when iterating the duality operation. Given an item C, we may use C* to denote the dual of C. The fundamental difference between the two families of dualities thus becomes the difference between case $C^{**} = C$ and the case $C^{**} \neq C$. I shall call the first form of duality an “elementary duality” and its complement a “non-elementary duality”.

Non-elementary duality is a fundamental aspect of the categorical analysis of ontology. For the sake of simplicity, here the term “categories” refers to both principles and categories in the strict sense (on the difference between categories and principles in Aristotle see Poli 2001, p. 59).

To show how broad is the phenomenon of categorical duality, suffice it to mention some of the best-known categorical pairs.

Besides pairs of categories with a recognized mathematical meaning, like finite/infinite and discrete/continuous, one easily finds many further categorical pairs that have figured prominently in the history of philosophy. For example:

- matter/form
- potential/act
- quality/quantity
- one/many
- identity/difference
- individual/universal
- part/whole

A moment’s reflection shows that the dualities that hold within these pairs of categories *are not elementary*: the form of a certain substance is the substance of another form different from the initial one; the act of a certain potential is the potential of a new act different from the initial one; and so on.

The categorical duality that holds in ontology (at least with regard to the pairs of categories listed above) is therefore not an elementary duality.

14. INTERMEZZO

The final part of this article exemplifies in formal terms (some of) the distinctions introduced in Table 2 above.

The story whose outcomes are described below began with a seminar on the *Tractatus* organized by Tadeusz Czeowski in Toru at the end of the 1950s. The seminar was attended by Bogusaw Wolniewicz, who recast its contents in original form. The results of this re-elaboration were set out in *Rzeczy i facty* [Things and facts], 1968, and in *Ontologia sytuacij* [Ontology of situations], 1985 (some of Wolniewicz's works in English are cited in the references). During the 1960s, Roman Suszko met Wolniewicz and read the manuscript of the former book. Thereafter he developed the so-called W-languages (W for Wittgenstein) from which derive the non-Fregean logics outlined below (see the references for bibliographical details).

Independently of the Polish logicians, Barwise and Perry developed a somewhat similar theory in *Situations and Attitudes*, As far as I know, a systematic comparison between the two perspectives has not yet been conducted.

15. SEMIOTIC PRELIMINARIES

As we know, for Frege there were only two ontological correlates of propositions: the True and the False. All true propositions denote the True, and all false prepositions denote the False. From an ontological point of view, if all true propositions denote exactly one and the same entity, then the underlying philosophical position is the absolute monism of facts.

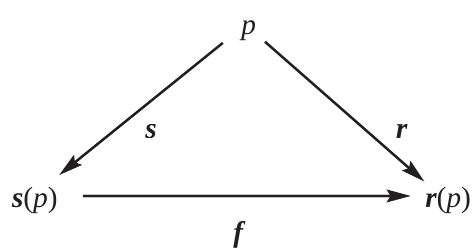
In what follows I shall seek to disprove what Suszko called "Frege's axiom": namely the assumption that there exist only two referents for propositions.

Frege's position on propositions was part of a more general view. Indeed, Frege adopted a *principle of homogeneity* (Perzanowski, 1992) according to which there are two fundamental categories of signs (*Bedeutungen* and truth-values) and two fundamental categories of senses (*Sinn* and *Gedanken*).

Both categories of signs (names and propositions) have sense and reference. The sense of a name is its *Sinn*, that way in which its referent is given, while the referent itself, the *Bedeutung*, is the object named by the name. As for propositions, their sense is the *Gedanke*, while their reference is their logical value.

Since the two semiotic triangles are entirely similar in structure, we need analyze only one of them: that relative to propositions.

Fig. 1

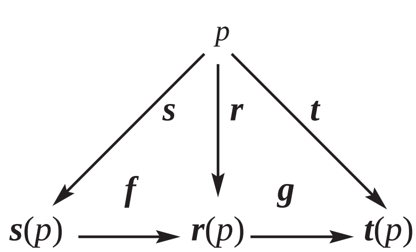


Here p is a proposition, $s(p)$ is the sense of p, and $r(p)$ is the referent of p. The functional composition states that $s(p)$ is the way in which p yields $r(p)$. The triangle has been drawn with the functions linking its vertexes explicitly shown. When the functions are composable, the triangle is said to commute, yielding $f(s(p)) = r(p)$, or $f \circ s(p) = r(p)$.

An interesting question now arises: is it possible to generalize the semiotic triangle? And if it is possible to do so, what is required?

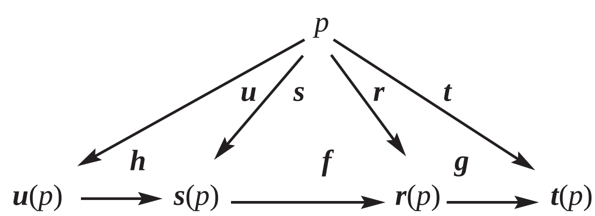
Useful here are the Husserlian distinctions set out in Table 2. A first reorganization and generalization of the semiotic triangle therefore involves an explicit differentiation between the truth-value assigning function and the referent assigning function. We thus have the following double semiotic triangle:

Fig. 2



where *r* stands for the referent assigning function and *t* for the truth-value assigning function. Again on the basis of Table 2, we may further extend the original semiotic triangle by also considering utterances:

Fig. 3



In order not to complicate the situation excessively, in what follows I shall deal solely with the right-hand side of the multi-triangle in Fig. 3, the one corresponding to what I have called the double semiotic triangle (Fig. 2).

Suszko uses the terms *logical valuations* for the procedures that assign truth-values, and *algebraic valuations* for those that assign referents. By arguing for the existence of only two referents, Frege ends up by collapsing logical and algebraic valuations together, thereby rendering them indistinguishable.

Having generalized the semiotic triangle into the double semiotic triangle, we must now address the following questions:

1. when do two propositions have the same truth value?
2. when do two propositions have the same referent?
3. when do two propositions have the same sense?

I shall distinguish among the three cases by talking respectively of *equivalence*, *identity* and *synonymy*. Sameness of logical value will be denoted by $\leftrightarrow$ (logical equivalence), while sameness of referent will be indicated with $\equiv$ (not to be confused with the equiform $\equiv$ used in § 7 to express indiscernibility) and sameness of sense (synonymy) by $\approx$. Two propositions are synonymous when they have the same sense:

$$(p \approx q) = 1 \text{ iff } (\boldsymbol{s}(p) = \boldsymbol{s}(q)) = 1.$$

Two propositions are identical when they have the same referent:

$$(p \equiv q) = 1 \text{ iff } (\boldsymbol{r}(p) = \boldsymbol{r}(q)) = 1.$$

Two propositions are equivalent when they have the same truth value:

$$(p \leftrightarrow q) = 1 \text{ iff } (\boldsymbol{t}(p) = \boldsymbol{t}(q)) = 1.$$

These various concepts are functionally connected as follows:

$$\boldsymbol{s}(p) = \boldsymbol{s}(q) \text{ implies } \boldsymbol{r}(p) = \boldsymbol{r}(q),$$
$$\boldsymbol{r}(p) = \boldsymbol{r}(q) \text{ implies } \boldsymbol{t}(p) = \boldsymbol{t}(q).$$

I shall now examine the identity connective in particular.

In general, the constraints that we impose on referents correspond to the *ontological assumptions* that characterize the theory. The most general logic of all is the one that imposes no restriction at all on $\boldsymbol{r}$ valuations. Just as Fregean logic recognizes only two referents so the most general logic recognizes a more than numerable set of them. Between these two extremes, of course, there are numerous intermediate cases. Pure non-Fregean logic is extremely weak, a chaos. If it is to yield something, it must be strengthened.

16. NON-FREGEAN LOGICS (NFLs)

In accordance with the Polish tradition, here by "logic" is not meant a set of theorems but a consequence relation generated by axioms and rules of inference (cf. Wojcicki 1988). In general terms, a formal language is composed of

- declarative variables;
- nominal and predicative variables;
- truth-functional connectives and auxiliary signs:
- quantifiers;
- identity connective;
- identity predicate.

Since here we are only concerned with propositional logics, our language will be much simpler. For example, there will be no nominal or predicative variables and consequently no identity predicate. Instead, unlike in the usual propositional calculuses, there will be quantifiers. The reason for this is obvious. In Fregean calculus, the quantifiers vary on the two sole situations constituted by the truth-values of truth and falsehood and have no role to perform. Instead, in a logic which admits a multiplicity of situations, the quantifiers have an effective function to perform. We shall also have an identity connective distinct from double implication. In the usual propositional calculuses, identity is obviously indistinguishable from double implication. Unlike the usual connectives, the identity connective is not truth-functional: in other words, it does not admit an extensional principle of composition.

The propositional component of NFLs is termed Sentential Calculus with Identity (**SCI**). Obviously, there is truth-functional bivalence in non-Fregean logics but no ontological bivalence.

As said, propositional identity will be denoted with the symbol ≡, and an equation will be called a formula of the type A ≡ B. Only *modus ponens* will be used as a derivation rule. In order to characterize the calculus, we need:

- axioms for the truth-functional connectives
- axioms for the quantifiers
- axioms for the identity connective.

For the sake of simplicity I shall omit the section on the quantifiers. In Ishii's (2000) version, the truth-functional axioms (**TFA**) of logic ***L*** are:

A1. $A \rightarrow (B \rightarrow A)$
A2. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$
A3. $A \wedge B \rightarrow A$
A4. $A \wedge B \rightarrow B$
A5. $A \rightarrow (B \rightarrow (A \wedge B))$
A6. $A \rightarrow (A \vee B)$
A7. $B \rightarrow (A \vee B)$
A8. $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$
A9. $A \rightarrow (\neg A \rightarrow B)$
A10. $\neg\neg A \rightarrow A$

The identity axioms (**IDA**) are now added to the truth-functional axioms:

A11. $A \equiv A$
A12. $(A \equiv B) \rightarrow (B \equiv A)$
A13. $(A \equiv B) \wedge (B \equiv C) \rightarrow (A \equiv C)$
A14. $(A \equiv B) \rightarrow (\neg A \equiv \neg B)$
A15. $(A \equiv B) \wedge (C \equiv D) \rightarrow (A \wedge C) \equiv (B \wedge D)$
A16. $(A \equiv B) \wedge (C \equiv D) \rightarrow (A \vee C) \equiv (B \vee D)$
A17. $(A \equiv B) \wedge (C \equiv D) \rightarrow (A \rightarrow C) \equiv (B \rightarrow D)$
A18. $(A \equiv B) \wedge (C \equiv D) \rightarrow (A \equiv C) \equiv (B \equiv D)$
A19. $(A \equiv B) \rightarrow (A \rightarrow B)$

The union of the truth-functional axioms **TFA** and of the identity axioms **IDA** will be termed the set of logical axioms and called **LA**.

17. PROPOSITIONAL CALCULUSES WITH IDENTITY (PCI)

The non-Fregean logics just outlined are very general. Yet, as Ishii (2000) points out, they are not entirely general. In order to obtain an effectively general situation we must abolish axioms A18 and A19, thereby relinquishing transitivity and reflexivity. Ishii has called the logic thus obtained Propositional Calculus with Identity (**PCI**).

PCI is so weak that it does not even comprise a substitution law $A \equiv B \rightarrow (G[A/p] \equiv G[B/p])$, which instead holds for **NFLs**. For this reason, **PCI** has all the credentials to perform the role of a general logic.

18. TRUTH VALUATION

The next step is to valuate, in perfectly routine manner, the truth-value of our propositions. Given a truth valuation, we immediately have a logical consequence dependent upon it, as well as the set of its tautologies called **TAUT**. In analytical terms: a function V from the set of formulas to the truth values {0,1} is a truth valuation iff:

$V(\neg A) = \neg V(A)$
$V(A \wedge B) = 1$ iff $V(A) = V(B) = 1$ (and so on, as usual, for $\vee$, $\rightarrow$, $\leftrightarrow$).
$V(A \equiv A) = 1$
$V(\neg A \equiv \neg B) = 1$ iff $V(A \equiv B) = 1$
$V([A \# C] \leftrightarrow [B \# D]) = 1$ only if $V(A \equiv B) = V(C \equiv D) = 1$, where # stands for anyone of the binary connectives $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and $\equiv$
$V(A \equiv B) = 0$ whenever $V(A) \neq V(B)$.

It follows immediately that **TAUT** is invariant and finite (and therefore decidable).

We divide truth functions into two groups: those that hold for the usual truth-functional connectives, and those that hold for the identity connective. The former group comprises the truth-functional tautologies (**TFT**), for example formulas like $p \vee \neg p$, $p \rightarrow p$, $\neg(p \wedge \neg p)$, ecc. This is obviously an invariant set.

Having distinguished the sub-set **TFT** from **TAUT**, the question naturally arises as to which formulas pertain to the complement of **TFT**. Here are some examples: $p \equiv p$, $(p \equiv q) \wedge (q \equiv r) \rightarrow (p \equiv r)$, $(p \equiv q) \rightarrow (q \equiv p)$, $(p \equiv q) \rightarrow (\neg p \equiv \neg q)$.

As Suszko points out, these formulas are entirely trivial because none of them can be an equation of the form $A \equiv B$ or of the form $\neg(A \equiv B)$. Nevertheless we also obtain a non-trivial result: the identity connective is tautologically symmetric, which implies that $(p \equiv q) \leftrightarrow (q \equiv p)$ is a tautology that pertains to the complement of **TFT**. By contrast, $(p \equiv q) \equiv (q \equiv p)$ is not even a tautology. Put in discursive terms, to say that "p is identical to q" is equivalent to "q is identical to p" is a tautology, while to say that "p is identical to q" and "q is identical to p" are identically the same statement is not a tautology.

There are numerous examples of the type just given. Because the equivalences that follow are in **TFT** they are also in **TAUT**:

- $\neg\neg(p) \leftrightarrow p$;
- $p \wedge p \leftrightarrow p$;
- $(p \vee q) \leftrightarrow \neg(p \rightarrow q)$;
- $(p \wedge q) \leftrightarrow \neg(\neg p \vee \neg q)$.

But the equations corresponding to them are not in **TFT**:

- $\neg\neg(p) \equiv p$;
- $p \wedge p \equiv p$;
- $(p \vee q) \equiv \neg(p \rightarrow q)$;
- $(p \wedge q) \equiv \neg(\neg p \vee \neg q)$.

This reiterates the difference between formal equivalence and identity of referents. In non-Fregean logics, an expression like $\neg(p \vee q \equiv \neg p \rightarrow q)$ is not a problem. If one decides to construe $p \vee q$ as an abbreviation of $p \rightarrow q$, then one has decided to adopt $p \vee q \equiv p \rightarrow q$ as axiom. The same applies to all abbreviations.

In general, the triviality of the logical theorems of the calculus confirms the fact that we are operating at an absolutely general level. In order to obtain meaningful theorems we must add some supplementary conditions, as we have seen.

19. EXTENSIONS

The extreme weakness of the logics discussed is further borne out by the fact that every adequate model of the consequence relation of NFLs – call it ***Cn*** – is more than countable, i.e. has the power of the continuum. For reasons of both technical manageability and ontological valuation, however, we need something stronger: namely extensions of ***Cn*** and in particular invariant extensions, i.e. ones closed under substitution.

An extension obtained by adding an additional set of axioms to **LA** is an “elementary extension”. The logics obtained by adding new axioms have some non-logical content, i.e. non-tautological assumptions of ontological significance.

The first and most natural step is therefore to consider the elementary extensions of ***Cn***.

Suszko has developed extensions of **SCI** able to represent Lukasiewicz’s three-valued system (interpreting $A \equiv B$ as $A \underline{\leftrightarrow} B$, where $\underline{\leftrightarrow}$ is Lukasiewicz’s three-value equivalence), and the modal systems **S4** and **S5** (interpreting $A \equiv B$ as $\Box(A \leftrightarrow B)$). Ishii (2000) has generalized the procedure, representing numerous other logics in opportune extensions of **PCI** (Corsi’s weak logic **F**, Girard’s linear logic **GL**, modal logics **K**, **KT**, **KB**, **K4**, **KD**, **K5).**

I shall restrict my discussion to the passage from ***Cn*** to ***CnF***, where ***CnF*** is the consequence relation that holds for Fregean logic.

The invariant theorem on which Fregean logic is based is given by any one of the following axioms:

$(A \leftrightarrow B) \rightarrow (A \equiv B)$
$(A \leftrightarrow B) \leftrightarrow (A \equiv B)$
$(A \leftrightarrow B) \equiv (A \equiv B)$
$(A \equiv B) \vee (A \equiv C) \vee (B \equiv C)$

I shall refer to any of these axioms as **aF**. Note that the third formula explicitly identifies $\leftrightarrow$ with $\equiv$. The two connectives thus become completely indistinguishable, $\equiv$ becomes truth-functional, and $\leftrightarrow$ has all the properties of the identity connective. It is not difficult to ascertain that **aF** is neither in **TFT** nor in **TAUT**.

The procedure briefly exemplified indicates that the general logic can be constrained by adding various groups of axioms relative to identity, i.e to the universe of the referents.

Both Frege’s axiom and its negation are logical theorems because both can be adopted by non-Fregean logics. The independence of Frege’s axiom was proved by Tarski in his doctoral thesis (1923), in which he explicitly compared it to Euclid's fifth postulate (Suszko, 1977, p. 379). Frege's logic (**FL**) is therefore a particular case of **NFL**.

20. CONCLUSION

The formal analysis of the previous sections has various aspects which should be explicitly mentioned.

The first and obvious one is that the ontological-formal restrictions imposed on the propositional fragment of ***NFL*** articulate the content of the identity connective. Hence identity is neither an empty concept nor a universal invariant.

The second aspect is that numerous ontologically interesting variations of logical systems seem to operate in the space between ***Cn*** and ***CnF***. It remains to be seen whether the extensions of ***Cn*** which do not lead to ***CnF*** are worth studying as well (probably not, but this cannot be taken for granted).

Thirdly, usual logical practice does not move in the same direction that we have followed (from ***Cn*** to ***CnF***). They usually start from classical logic and extend it in various ways (for example by adding new operators, as in modal logics). This raises the problem of the connections between the two strategies: the one that sees classical logic as its point of departure and the one that sees it as its point of arrival. From an ontological point of view, the latter approach seems perfectly sensible, because it articulates the spectrum of positions that lead from indeterminate chaos to monism. The ontological significance of the other approach is not clear (so that it is not surprising if the ontological sense of the semantics of possible worlds is obscure). Whatever the case may be, the problem of the relationship between what comes before and what comes after Fregean logic (well known since Gödel's mapping between intuitionist logic and **S4**) is still open, from an ontological viewpoint.

The fourth aspect concerns the problem of the duality between identity and difference. **PCI** logics can be understood from the point of view of variations of identity (point 1 above). But what happens if we decide to use difference instead of identity? Intuitionist logic and mathematics, for example, seem more easily constructable on the basis of the relation of *apartness*, which is a type of difference (cf. van Dalen, 1994). We know that this yields a theory of order more sophisticated than the classical one. Is difference perhaps richer than identity?

Fifthly, the logics discussed have a classical basis and they terminate in classical logic. What happens if we try to adopt an intuitionist basis?

Finally, the items of the universe can be studied in various ways. The two that have proved most useful are *decomposition* and *parameterization*. The decomposition method presupposes that it is possible to inspect the item, and therefore that it is at least partly amenable to investigation. Ideally, the *decomposition* method works perfectly when the procedure is reversible and the whole can be reconstructed from its parts. The principle of truth-functional composition is a perfect formal codification of this ideal situation.

By contrast, the *parameterization* method treats the item as a black box which cannot be opened to see what is inside. But it is still possible to study the item by analyzing its responses to changes in external conditions (which in this case function as parameters). The most banal example is that of parameterization with respect to time. In this case, the expression $T \rightarrow O$ can be used to indicate study of item O with respect to the parameter "time". The study of identity (and the correlated study of difference) briefly outlined above belongs to this second method of analysis.